Superconducting properties of carbon nanotubes

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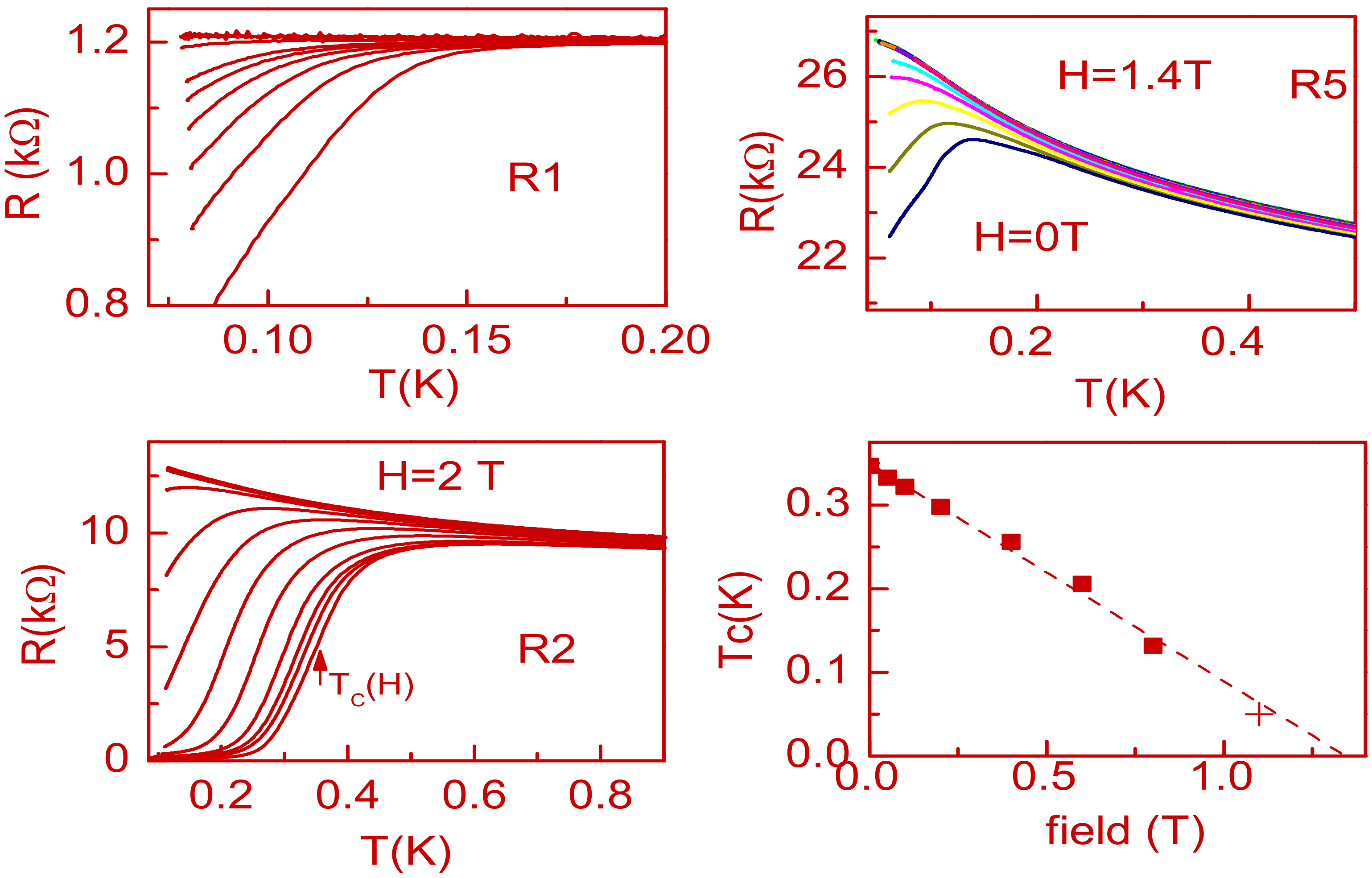
Overview

- Superconductivity in ropes of nanotubes
  - Attractive interactions via phonon exchange
  - Effective low energy theory for superconductivity
  - Quantum phase slips, finite resistance in the superconducting state
- Josephson current through a short nanotube
  - Supercurrent through correlated quantum dot via Quantum Monte Carlo simulations
  - Kondo physics versus π-junction, universality

Classification of carbon nanotubes

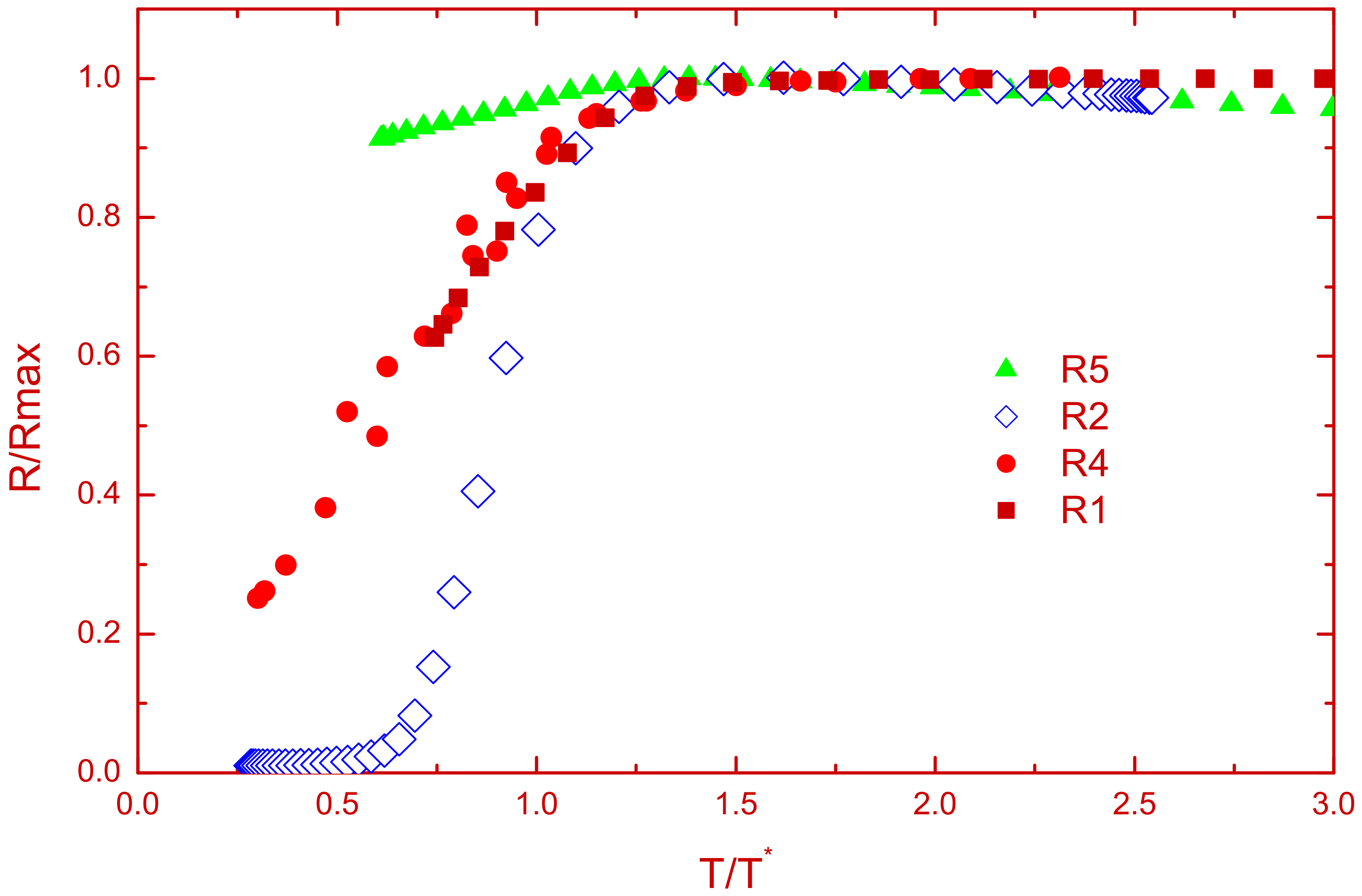
- Single-wall nanotubes (SWNTs):
  - One wrapped graphite sheet
  - Typical radius 1 nm, lengths up to several mm
- Ropes of SWNTs:
  - Triangular lattice of individual SWNTs (typically up to a few 100)
- Multi-wall nanotubes (MWNTs):
  - Russian doll structure, several inner shells
  - Outermost shell radius about 5 nm

Superconductivity in ropes of SWNTs: Experimental results



Kasumov et al., PRB 2003

Experimental results II



Kasumov et al., PRB 2003

Continuum elastic theory of a SWNT: Acoustic phonons

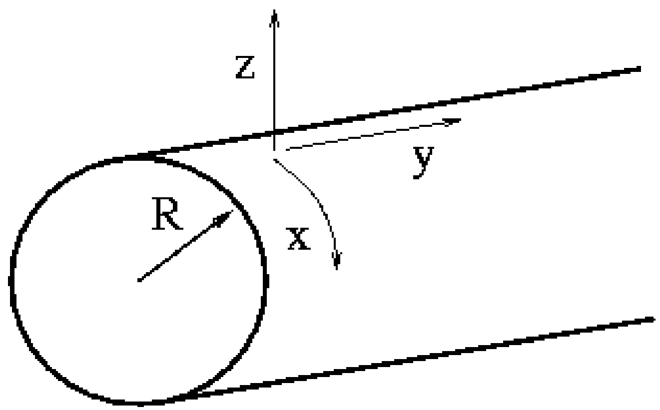
De Martino & Egger, PRB 2003

- Displacement field: $\vec{u}(x,y) = (u_x, u_y, u_z)$
- Strain tensor:

$$u_{yy} = \partial_y u_y$$

$$u_{xx} = \partial_x u_x + u_z / R$$

$$2u_{xy} = \partial_y u_x + \partial_x u_y$$



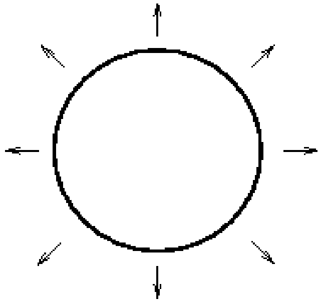
- Elastic energy density:

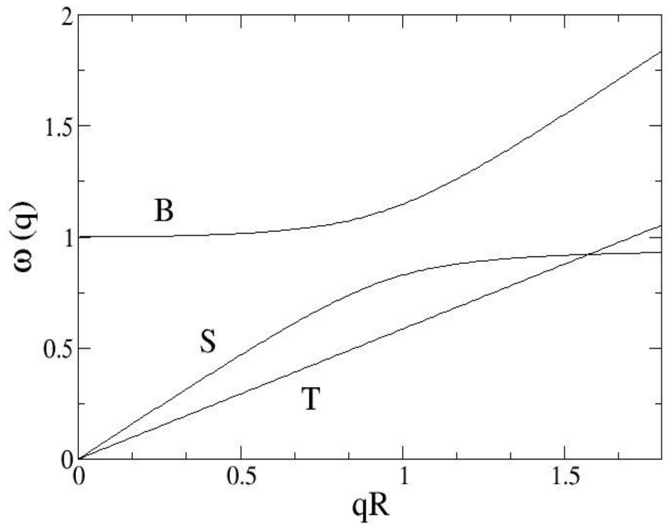
$$U(\vec{u}) = \frac{B}{2}\left(u_{xx} - u_{yy}\right)^2 + \frac{\mu}{2}\left(\left(u_{xx} - u_{yy}\right)^2 + 4u_{xy}^2\right)$$

Suzuura & Ando, PRB 2002

Normal mode analysis

- Breathing mode

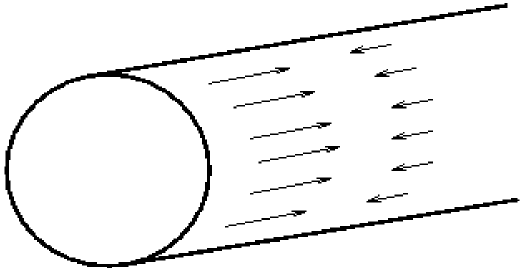




$$\omega_B = \sqrt{\frac{B+\mu}{MR^2}} \approx \frac{0.14}{R}\,\mathrm{eV}\,\mathring{A}$$

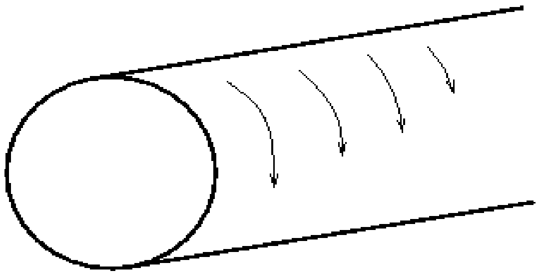
- Stretch mode

$$v_S = \sqrt{4B\mu / M(B+\mu)} \approx 2\times10^4\ \mathrm{m/s}$$



- Twist mode

$$v_T = \sqrt{\mu / M} \approx 1.2\times10^4\ \mathrm{m/s}$$



Electron-phonon coupling

- Main contribution from deformation potential

$$V(x,y) = \alpha\left(u_{xx} + u_{yy}\right) \qquad \alpha \approx 20-30\,\mathrm{eV}$$

couples to electron density

$$H_{el-ph} = \int dxdy\ V\rho$$

- Other electron-phonon couplings small, but potentially responsible for Peierls distortion
- Effective electron-electron interaction generated via phonon exchange (integrate out phonons)

SWNT as Luttinger liquid

Egger & Gogolin; Kane et al., PRL 1997
De Martino & Egger, PRB 2003

- Low-energy theory of SWNT: Luttinger liquid
- Coulomb interaction: $g = g_0 \leq 1$
- Breathing-mode phonon exchange causes attractive interaction: $$g = \frac{g_0}{\sqrt{1 - g_0^2 R_B / R}}$$

For (10,10) SWNT:
$g \approx 1.3 > 1$

$$R_B = \frac{2\alpha^2}{\pi^2 v_F (B + \mu)} \approx 0.24nm$$

- Wentzel-Bardeen singularity: very thin SWNT

Superconductivity in ropes

De Martino & Egger, cond-mat/0308162

Model:

$$H = \sum_{i=1}^{N} H_{Lutt}^{(i)} - \sum_{ij} \Lambda_{ij} \int dy\, \Theta_i^* \Theta_j$$

- Attractive electron-electron interaction within each of the *N* metallic SWNTs
- Arbitrary Josephson coupling matrix, keep only singlet on-tube Cooper pair field $\Theta_i(y,\tau)$
- Single-particle hopping negligible

Maarouf, Kane & Mele, PRB 2003

Order parameter for nanotube rope superconductivity

- Hubbard Stratonovich transformation: complex order parameter field

$$\Delta_i(y,\tau) = |\Delta_i| e^{i\Phi_i}$$

  to decouple Josephson terms

- Integration over Luttinger liquid fields gives formally exact effective (Euclidean) action:

$$S = \sum_{ij,y\tau} \Delta_i^* \Lambda_{ij}^{-1} \Delta_j - \ln \left\langle e^{-Tr\left(\Delta^* \Theta + \Theta^* \Delta\right)} \right\rangle_{Lutt}$$

Quantum Ginzburg Landau (QGL) theory

- 1D fluctuations suppress superconductivity
- Systematic cumulant & gradient expansion:
  Expansion parameter $|\Delta|/2\pi T$
- QGL action, coefficients from full model

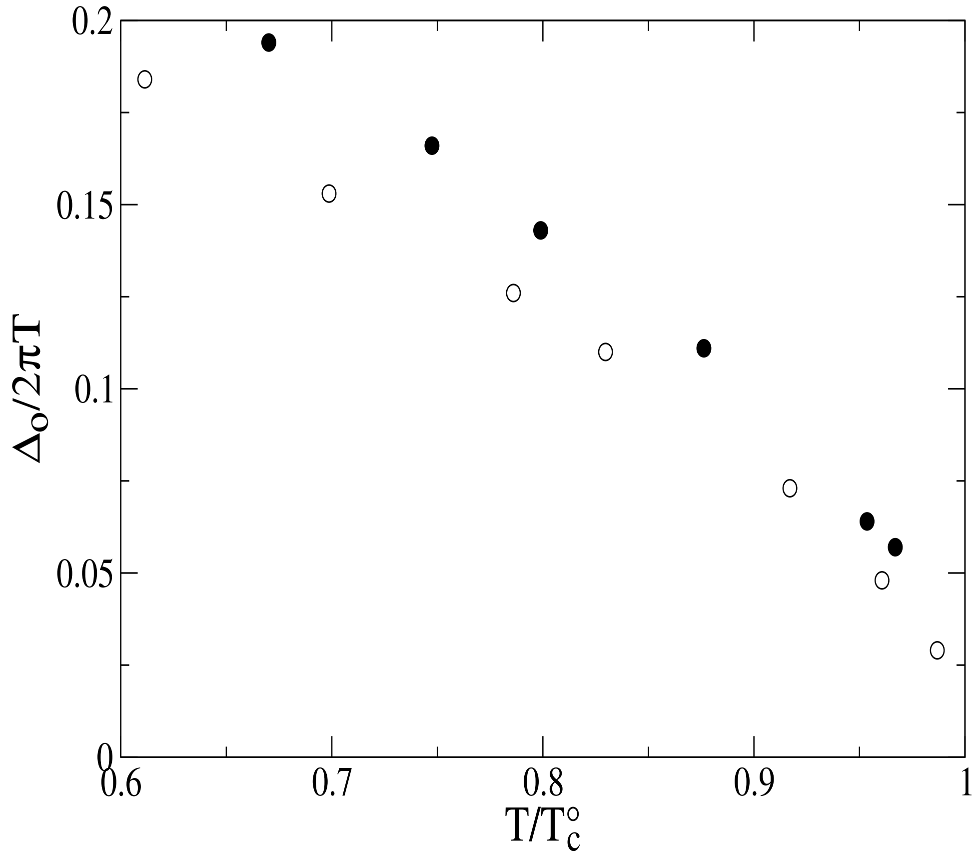
$$S = Tr\left\{\left(\Lambda_1^{-1} - A\right)|\Delta|^2 + B|\Delta|^4\right\} +$$
$$+ Tr\left\{C|\partial_y \Delta|^2 + D|\partial_\tau \Delta|^2\right\} +$$
$$+ Tr \sum_{ij} \Delta_i^* \left(\Lambda_{ij}^{-1} - \Lambda_1^{-1}\right)\Delta_j$$

Amplitude of the order parameter

- Mean-field transition at

$$A\left(T_c^{\,0}\right) = \Lambda_1$$

- For lower *T*, amplitudes are finite, with gapped fluctuations
- Transverse fluctuations irrelevant for $N \leq 100$
- QGL accurate down to very low T



Low-energy theory: Phase action

- Fix amplitude at mean-field value: Low-energy physics related to phase fluctuations

$$S = \frac{\mu}{2\pi}\int dy\,d\tau\left[c_s^{-1}\left(\partial_\tau\Phi\right)^2 + c_s\left(\partial_y\Phi\right)^2\right]$$

- Rigidity

$$\mu(T) = N\nu\left[1 - \left(T/T_c^0\right)^{(g-1)/2g}\right]$$

$\nu \approx 1$ from QGL, but also influenced by dissipation or disorder

Quantum phase slips: Kosterlitz-Thouless transition to normal state

- Superconductivity can be destroyed by vortex excitations: Quantum phase slips (QPS)
- Local destruction of superconducting order allows phase to slip by 2π
- QPS proliferate for $\mu(T) \leq 2$
- True transition temperature

$$T_c = T_c^0 \left[1 - \frac{2}{N\nu} \right]^{2g/(g-1)} \approx 0.1 \ldots 0.5K$$

Resistance in superconducting state

- QPS-induced resistance
- Perturbative calculation, valid well below transition:

$$\frac{R(T)}{R(T_c)} = \left(T \big/ T_c \right)^{2\mu(T)-3} \frac{\int_0^\infty du \frac{1}{1+u^2} \left| \frac{\Gamma\left(\mu/2 + iuT_L/2T\right)}{\Gamma\left(\mu/2\right)} \right|^4}{\int du \frac{1}{1+u^2} \left| \frac{\Gamma\left(\mu/2 + iuT_L/2T_c\right)}{\Gamma\left(\mu/2\right)} \right|^4}$$

$$T_L = c_s \big/ \pi L$$

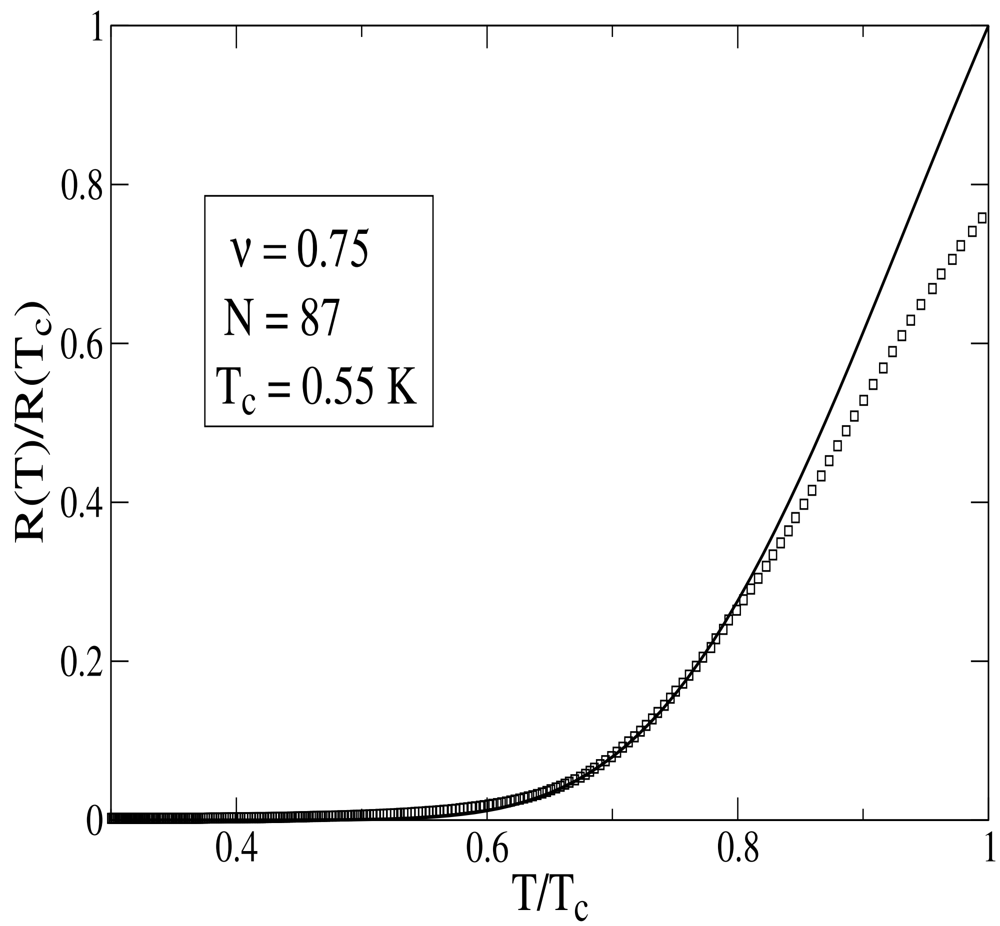
Comparison to experiment

- Resistance below transition allows detailed comparison to Orsay experiments
- Free parameters of the theory:
  - Interaction parameter, taken as $g = 1.3$
  - Number *N* of metallic SWNTs, known from residual resistance (contact resistance)
  - Josephson matrix (only largest eigenvalue needed), known from transition temperature
  - Only one fit parameter remains: $\nu \approx 1$

Comparison to experiment: Sample R2

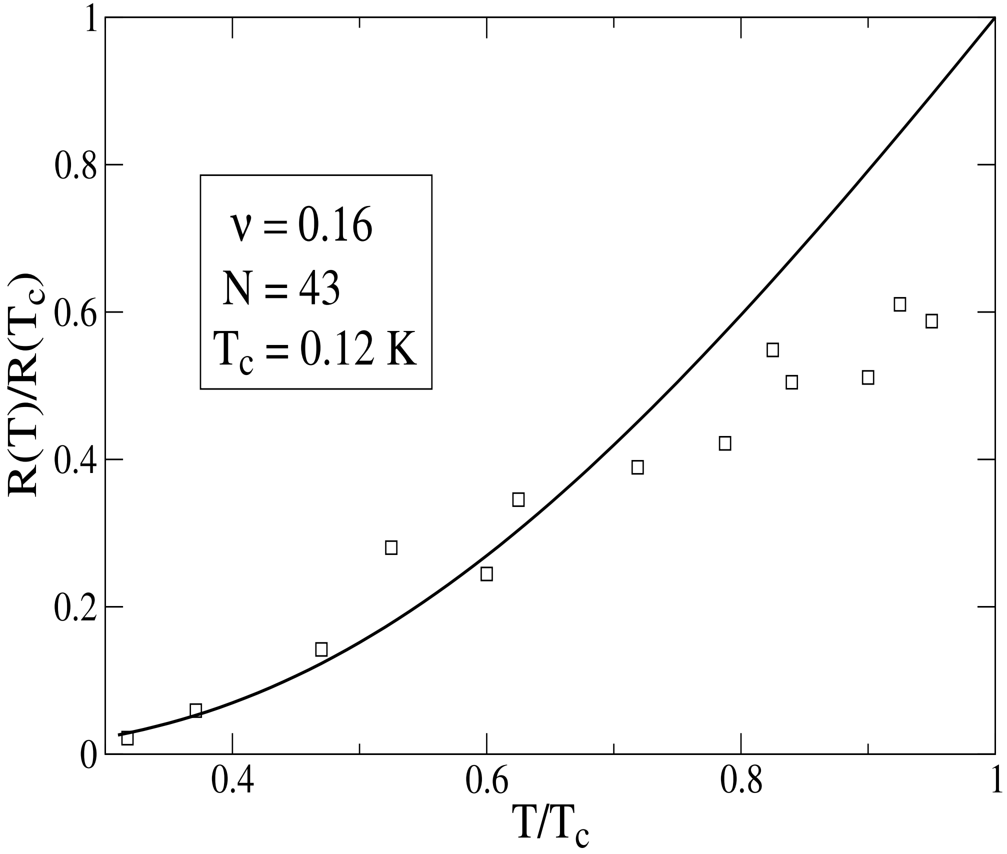
Nice agreement

- Fit parameter near 1
- Rounding near transition is not described by theory
- Quantum phase slips → low-temperature resistance
- Thinnest known superconductors

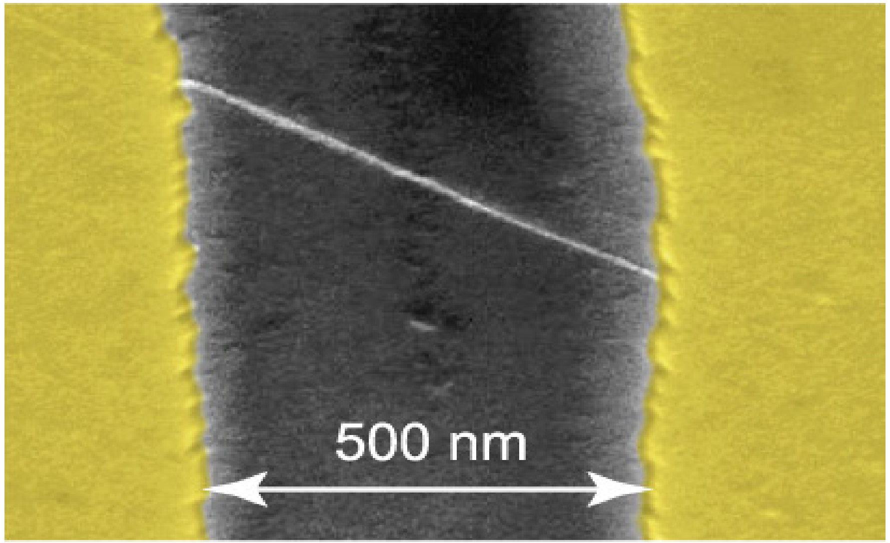


Comparison to experiment: Sample R4

- Again good agreement but more noise in experimental data
- Fit parameter now smaller than 1, dissipative effects
- Ropes of carbon nanotubes thus allow to observe quantum phase slips



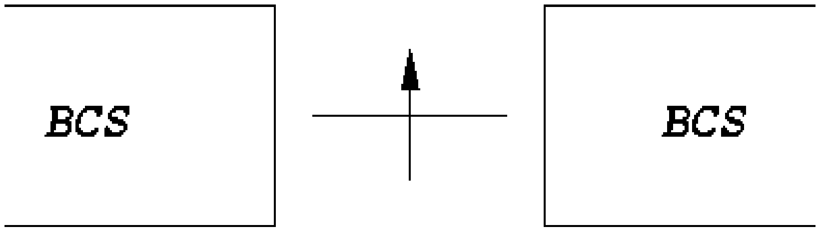
Josephson current through short tube



Buitelaar, Schönenberger et al., PRL 2002, 2003

- Short MWNT acts as (interacting) quantum dot
- Superconducting reservoirs: Josephson current, Andreev conductance, proximity effect ?
- Tunable properties (backgate), study interplay superconductivity $\leftrightarrow$ dot correlations

Model



- Short MWNT at low T: only a single spin-degenerate dot level is relevant
- Anderson model (symmetric)

$$H = H_{dot} + H_{coup} + H_{BCS}$$

$$H_{dot} = \varepsilon_0 \left(n_\uparrow + n_\downarrow \right) + U n_\uparrow n_\downarrow$$

- Free parameters:
  - Superconducting gap Δ, phase difference across dot Φ
  - Charging energy U, with gate voltage tuned to single occupancy: $\varepsilon_0 = -U/2$
  - Hybridization Γ between dot and BCS leads

Supercurrent through nanoscale dot

- How does correlated quantum dot affect the DC Josephson current?
  - Non-magnetic dot: Standard Josephson relation
    $$I(\phi) = I_c \sin\phi$$
  - Magnetic dot - Perturbation theory in Γ gives
    π-junction: $I_c < 0$ *Kulik, JETP 1965*
  - Interplay Kondo effect – superconductivity?
- Universality? Does only ratio Δ / T_K matter?

$$T_K \approx 0.2\sqrt{\Gamma U}\, e^{-\pi U / 4\Gamma}$$

Kondo temperature

Quantum Monte Carlo approach: Hirsch-Fye algorithm for BCS leads

- Discretize imaginary time in stepsize δ_τ
- Discrete Hubbard-Stratonovich transformation
  $\rightarrow$ Ising field $s_i = s(\tau_i) = \pm 1$ decouples Hubbard-U
- Effective coupling strength: $\cosh\lambda = e^{U\delta_\tau/2}$
- Trace out lead & dot fermions $\rightarrow$ self-energies

$$I(\phi) = \frac{1}{Z}\sum_{\{s_i\}} \det[\partial_\tau + \Sigma + \lambda s]\,\mathrm{Tr}\left([\partial_\tau + \Sigma + \lambda s]^{-1}\Sigma_J\right)$$

- Now stochastic sampling of Ising field

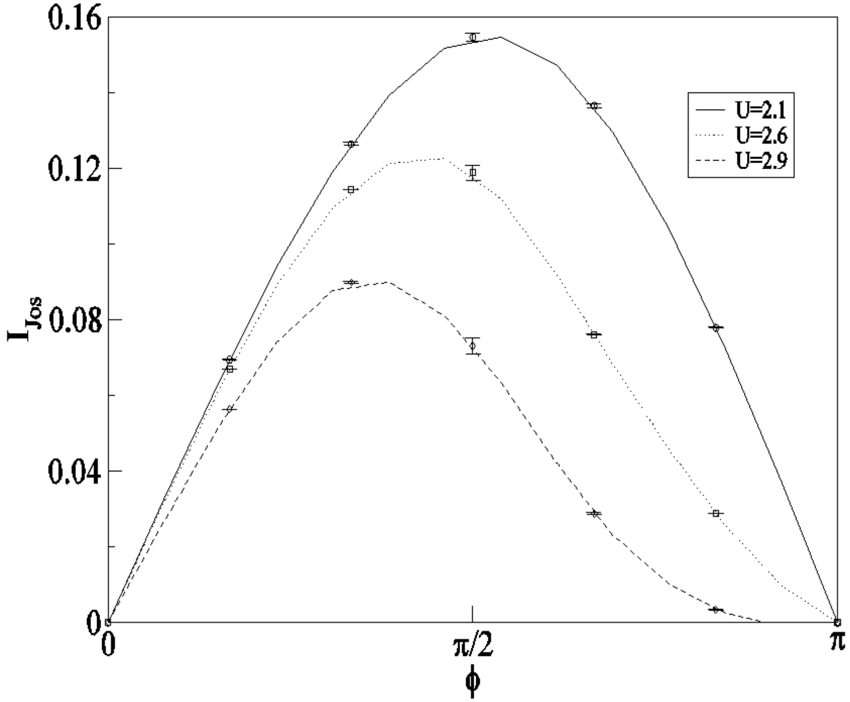
QMC approach

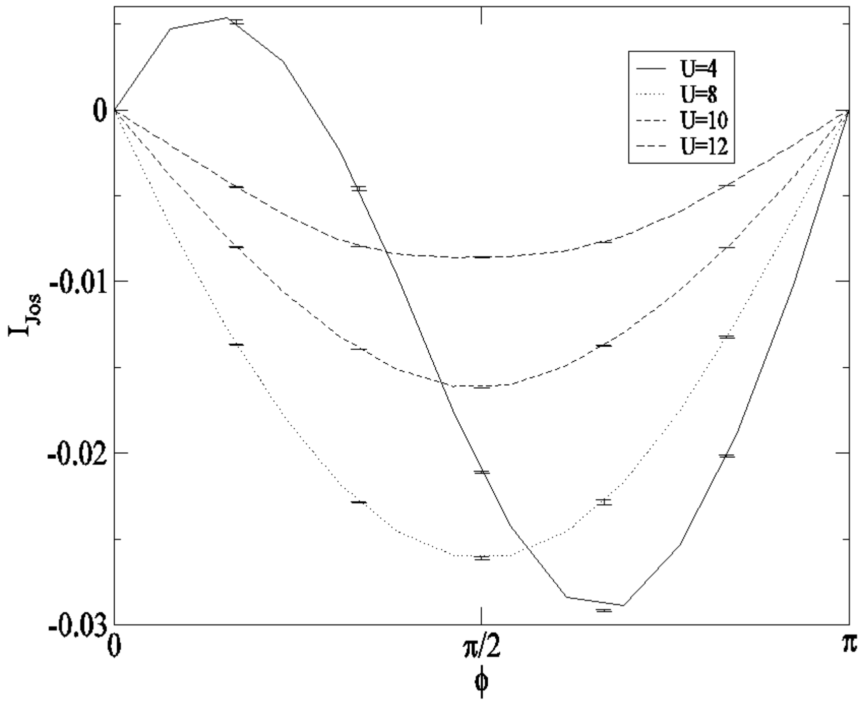
Siano & Egger

- Stochastic sampling of Ising paths
- Discretization error can be eliminated by extrapolation
- Numerically exact results
- Check: Perturbative results are reproduced
- Low temperature, close to T=0 limit
- Computationally intensive

Transition to π junction

$$\Delta = 1, T/\Delta = 0.1, \Gamma/\Delta = 1$$

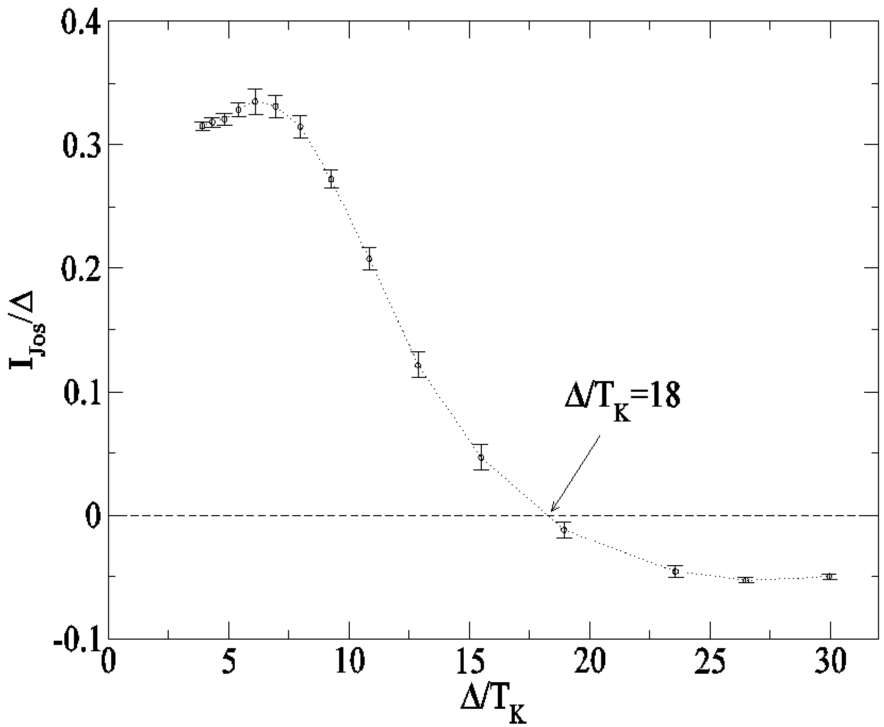




Kondo regime to π junction crossover

- Universality: Instead of Anderson parameters, everything controlled by ratio Δ / T_K
- Kondo regime has large Josephson current
  Glazman & Matveev, JETP 1989
- Crossover to π junction at surprisingly large

$$\Delta / T_K \approx 18$$



Conclusions

- Ropes of nanotubes exhibit intrinsic superconductivity, thinnest superconducting wires known
- Low-temperature resistance allows to detect quantum phase slips in a clear way
- Josephson current through short nanotube: Interplay between Kondo effect, superconductivity, and π junction