Superconducting properties of carbon nanotubes

Reinhold Egger
Institut für Theoretische Physik
Heinrich-Heine Universität Düsseldorf
A. De Martino, F. Siano
Overview

- Superconductivity in ropes of nanotubes
  - Attractive interactions via phonon exchange
  - Effective low energy theory for superconductivity
  - Quantum phase slips, finite resistance in the superconducting state

- Josephson current through a short nanotube
  - Supercurrent through correlated quantum dot via Quantum Monte Carlo simulations
  - Kondo physics versus $\pi$-junction, universality
Classification of carbon nanotubes

- **Single-wall nanotubes (SWNTs):**
  - One wrapped graphite sheet
  - Typical radius 1 nm, lengths up to several mm

- **Ropes of SWNTs:**
  - Triangular lattice of individual SWNTs (typically up to a few 100)

- **Multi-wall nanotubes (MWNTs):**
  - Russian doll structure, several inner shells
  - Outermost shell radius about 5 nm
Superconductivity in ropes of SWNTs: Experimental results

Kasumov et al., PRB 2003
Experimental results II

Kasumov et al., PRB 2003
Continuum elastic theory of a SWNT: Acoustic phonons

- Displacement field: \( \vec{u}(x, y) = (u_x, u_y, u_z) \)

- Strain tensor:
  \[
  u_{yy} = \partial_y u_y \\
  u_{xx} = \partial_x u_x + \frac{u_z}{R} \\
  2u_{xy} = \partial_y u_x + \partial_x u_y
  \]

- Elastic energy density:
  \[
  U(\vec{u}) = \frac{B}{2} (u_{xx} - u_{yy})^2 + \frac{\mu}{2} \left( (u_{xx} - u_{yy})^2 + 4u_{xy}^2 \right)
  \]

De Martino & Egger, PRB 2003
Suzuura & Ando, PRB 2002
Normal mode analysis

- **Breathing mode**
  \[ \omega_B = \sqrt{\frac{B + \mu}{MR^2}} \approx \frac{0.14}{R} \text{ eV Å} \]

- **Stretch mode**
  \[ \nu_S = \sqrt{\frac{4B\mu}{M(B + \mu)}} \approx 2 \times 10^4 \text{ m/s} \]

- **Twist mode**
  \[ \nu_T = \sqrt{\frac{\mu}{M}} \approx 1.2 \times 10^4 \text{ m/s} \]
Electron-phonon coupling

- Main contribution from deformation potential

\[ V(x, y) = \alpha (u_{xx} + u_{yy}) \quad \alpha \approx 20 - 30 \text{ eV} \]

couples to electron density

\[ H_{el-ph} = \int dx dy \ V \rho \]

- Other electron-phonon couplings small, but potentially responsible for Peierls distortion

- Effective electron-electron interaction generated via phonon exchange (integrate out phonons)
SWNT as Luttinger liquid

- Low-energy theory of SWNT: Luttinger liquid
- Coulomb interaction: \( g = g_0 \leq 1 \)
- Breathing-mode phonon exchange causes **attractive** interaction:
  \[
  g = \frac{g_0}{\sqrt{1 - g_0^2 R_B / R}}
  \]
  For (10,10) SWNT:
  \[
  g \approx 1.3 > 1
  \]
- Wentzel-Bardeen singularity: very thin SWNT
Superconductivity in ropes

De Martino & Egger, cond-mat/0308162

Model:

\[ H = \sum_{i=1}^{N} H_{\text{Lutt}}^{(i)} - \sum_{ij} \Lambda_{ij} \int dy \Theta^*_i \Theta_j \]

- Attractive electron-electron interaction within each of the \( N \) metallic SWNTs
- Arbitrary Josephson coupling matrix, keep only singlet on-tube Cooper pair field \( \Theta_i(y, \tau) \)
- Single-particle hopping negligible

Maarouf, Kane & Mele, PRB 2003
Order parameter for nanotube rope superconductivity

- Hubbard Stratonovich transformation: complex order parameter field
  \[ \Delta_i(y, \tau) = |\Delta_i|e^{i\Phi_i} \]
  to decouple Josephson terms

- Integration over Luttinger liquid fields gives formally exact effective (Euclidean) action:

\[
S = \sum_{ij, y \tau} \Delta_i^* \Lambda_{ij}^{-1} \Delta_j - \ln \left\langle e^{-Tr(\Delta^* \Theta + \Theta^* \Delta)} \right\rangle_{\text{Lutt}}
\]
Quantum Ginzburg Landau (QGL) theory

- 1D fluctuations suppress superconductivity
- **Systematic cumulant & gradient expansion:**
  Expansion parameter $|\Delta| / 2\pi T$
- QGL action, coefficients from full model

\[
S = Tr \left\{ \left( \Lambda_1^{-1} - A \right) |\Delta|^2 + B |\Delta|^4 \right\} + \\
+ Tr \left\{ C |\partial_y \Delta|^2 + D |\partial_\tau \Delta|^2 \right\} + \\
+ Tr \sum_{ij} \Delta^* \left( \Lambda_{ij}^{-1} - \Lambda_1^{-1} \right) \Delta_j
\]
Amplitude of the order parameter

- Mean-field transition at
  \[ A \left( T_c^0 \right) = \Lambda_1 \]

- For lower \( T \), amplitudes are finite, with gapped fluctuations

- Transverse fluctuations irrelevant for \( N \leq 100 \)

- QGL accurate down to very low \( T \)
Low-energy theory: Phase action

- Fix amplitude at mean-field value: Low-energy physics related to phase fluctuations

\[ S = \frac{\mu}{2\pi} \int dy d\tau \left[ c_s^{-1} (\partial_\tau \Phi)^2 + c_s (\partial_y \Phi)^2 \right] \]

- Rigidity

\[ \mu(T) = N \nu \left[ 1 - \left( \frac{T}{T_c^0} \right)^{(g-1)/2g} \right] \]

\( \nu \approx 1 \) from QGL, but also influenced by dissipation or disorder
Quantum phase slips: Kosterlitz-Thouless transition to normal state

- Superconductivity can be destroyed by vortex excitations: Quantum phase slips (QPS)
- Local destruction of superconducting order allows phase to slip by $2\pi$
- QPS proliferate for $\mu(T) \leq 2$
- True transition temperature

$$T_c = T_c^0 \left[1 - \frac{2}{N\nu}\right]^{2g/(g-1)} \approx 0.1...0.5K$$
Resistance in superconducting state

- QPS-induced resistance
- Perturbative calculation, valid well below transition:

\[
\frac{R(T)}{R(T_c)} = \left( \frac{T}{T_c} \right)^{2\mu(T)-3} \int_0^\infty du \frac{1}{1+u^2} \left| \frac{\Gamma(\mu/2 + iuT_L / 2T)}{\Gamma(\mu/2)} \right|^4 \int_0^\infty du \frac{1}{1+u^2} \left| \frac{\Gamma(\mu/2 + iuT_L / 2T_c)}{\Gamma(\mu/2)} \right|^4
\]

\[
T_L = \frac{c_s}{\pi L}
\]
Comparison to experiment

- Resistance below transition allows detailed comparison to Orsay experiments
- Free parameters of the theory:
  - Interaction parameter, taken as $g = 1.3$
  - Number $N$ of metallic SWNTs, known from residual resistance (contact resistance)
  - Josephson matrix (only largest eigenvalue needed), known from transition temperature
  - Only one fit parameter remains: $\nu \approx 1$
Comparison to experiment: Sample R2

Nice agreement

- Fit parameter near 1
- Rounding near transition is not described by theory
- Quantum phase slips → low-temperature resistance
- Thinnest known superconductors

\[ R(T)/R(T_c) \]

\[ T_c = 0.55 \text{ K} \]

\[ \nu = 0.75 \]

\[ N = 87 \]
Comparison to experiment: Sample R4

- Again good agreement but more noise in experimental data
- Fit parameter now smaller than 1, dissipative effects
- Ropes of carbon nanotubes thus allow to observe quantum phase slips
Josephson current through short tube

- Short MWNT acts as (interacting) quantum dot
- Superconducting reservoirs: Josephson current, Andreev conductance, proximity effect?
- Tunable properties (backgate), study interplay superconductivity ↔ dot correlations

Buitelaar, Schönenberger et al., PRL 2002, 2003
Model

- Short MWNT at low T: only a single spin-degenerate dot level is relevant
- **Anderson model**
  
  \[ H = H_{\text{dot}} + H_{\text{coup}} + H_{\text{BCS}} \]

  (symmetric)

- **Free parameters:**
  
  - Superconducting gap \( \Delta \), phase difference across dot \( \Phi \)
  - Charging energy \( U \), with gate voltage tuned to single occupancy: \( \varepsilon_0 = -U / 2 \)
  - Hybridization \( \Gamma \) between dot and BCS leads
Supercurrent through nanoscale dot

- How does correlated quantum dot affect the DC Josephson current?
  - Non-magnetic dot: Standard Josephson relation
    \[ I(\phi) = I_c \sin \phi \]
  - Magnetic dot - Perturbation theory in \( \Gamma \) gives
    \[ \pi \text{-junction: } I_c < 0 \quad \text{Kulik, JETP 1965} \]
  - Interplay Kondo effect – superconductivity?
- Universality? Does only ratio \( \frac{\Delta}{T_K} \) matter?

\[
T_K \approx 0.2\sqrt{\Gamma U} e^{-\pi U / 4\Gamma}
\]

Kondo temperature
Quantum Monte Carlo approach:
Hirsch-Fye algorithm for BCS leads

- Discretize imaginary time in stepsize $\delta_{\tau}$
- Discrete Hubbard-Stratonovich transformation $\rightarrow$ Ising field $s_i = s(\tau_i) = \pm 1$ decouples Hubbard-$U$
- Effective coupling strength: $\cosh \lambda = e^{U\delta_{\tau}/2}$
- Trace out lead & dot fermions $\rightarrow$ self-energies

\[
I(\phi) = \frac{1}{Z} \sum_{\{s_i\}} \det[\partial_{\tau} + \Sigma + \lambda s] \text{Tr} \left([\partial_{\tau} + \Sigma + \lambda s]^{-1} \Sigma_j \right)
\]

- Now stochastic sampling of Ising field
QMC approach

- Stochastic sampling of Ising paths
- Discretization error can be eliminated by extrapolation
- Numerically exact results
- Check: Perturbative results are reproduced
- Low temperature, close to T=0 limit
- Computationally intensive
Transition to $\pi$ junction

$\Delta = 1, T/\Delta = 0.1, \Gamma/\Delta = 1$
Kondo regime to $\pi$ junction crossover

- **Universality**: Instead of Anderson parameters, everything controlled by ratio $\Delta/T_K$
- **Kondo regime** has large Josephson current
  
  *Glazman & Matveev, JETP 1989*
- **Crossover** to $\pi$ junction at surprisingly large $\Delta/T_K \approx 18$
Conclusions

- Ropes of nanotubes exhibit intrinsic superconductivity, **thinnest superconducting wires known**
- Low-temperature resistance allows to detect **quantum phase slips** in a clear way
- Josephson current through short nanotube: Interplay between Kondo effect, superconductivity, and $\pi$ junction