Transport in interacting disordered nanotubes

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Overview

After introduction:

- Nonlinear magnetotransport in chiral interacting SWNTs (A. De Martino, A. Tsvelik)
- Crossover from Luttinger liquid to Altshuler-Aronov diffusive corrections in MWNTs (C. Mora, A. Altland)

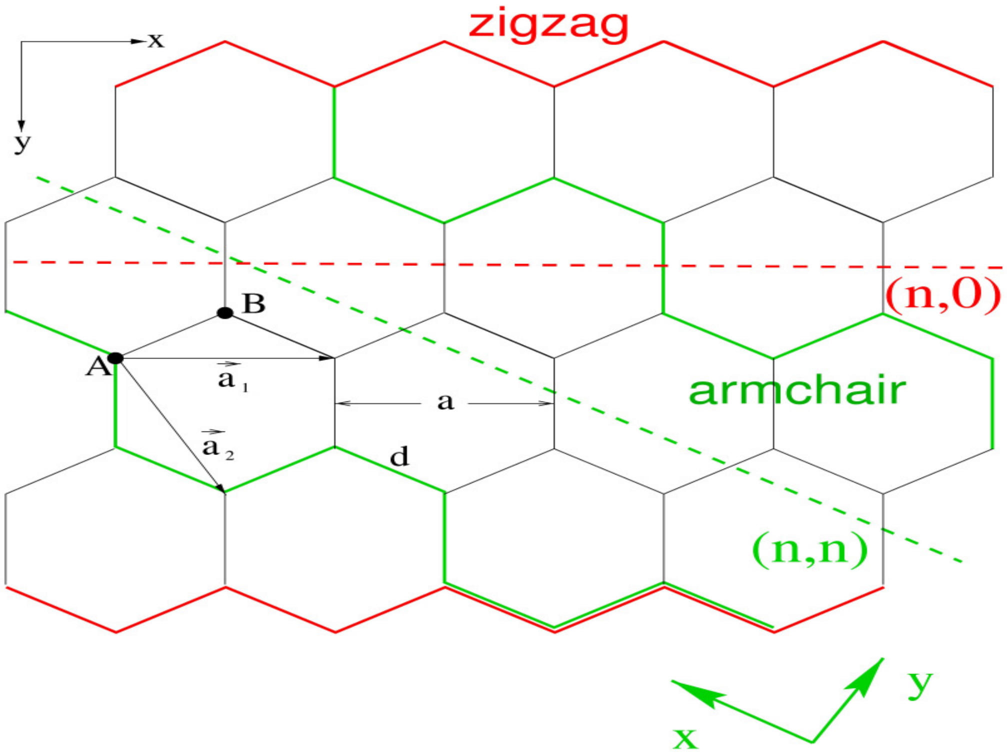
Wrapped 2D graphene sheet

Basis contains two atoms

$$a = \sqrt{3}d\,, d = 0.14nm$$

(n,m) indices: wrapping of sheet onto cylinder

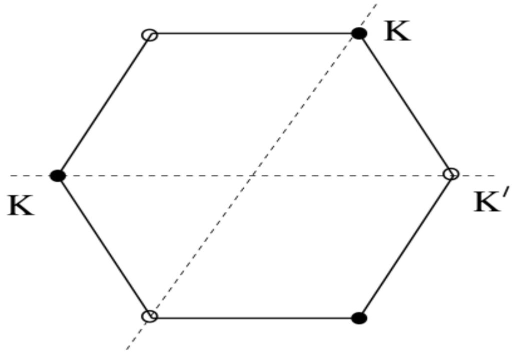
Chiral angle θ: defined with respect to zigzag (n,0) tube



Band structure: Graphene

Exactly two independent corner points K, K´ in first Brillouin zone.

Band structure: valence and conduction bands touch at corner points *(E=0)*, these are the Fermi points in graphene



- Lowest-order k•p scheme: Dirac light cone dispersion
- Deviations at higher energies: trigonal warping

$$E(\vec{q}) = v|\vec{q}|$$

$$\vec{q} = \vec{k} - \vec{K}$$

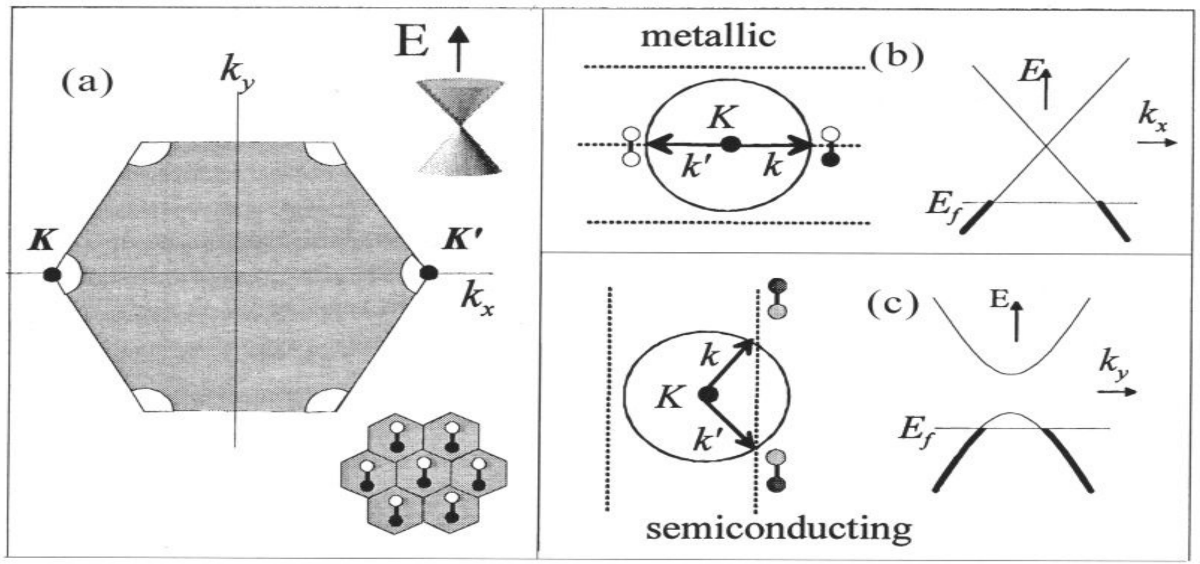
$$v = 8\times10^{5}\, m/\sec$$

Periodic boundary conditions: SWNTs

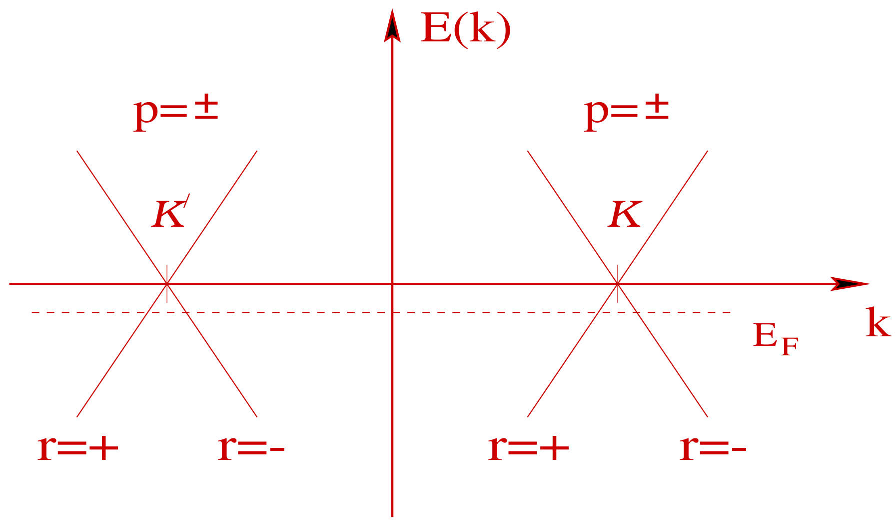
Transverse momentum must be quantized

Nanotube metallic only if K point has allowed transverse momentum

gives necessary condition: *2n+m = 3 x* integer



Metallic SWNTs



- Transverse momentum quantization: keep only $k_y = 0$
- Ideal 1D quantum wire: 2 spin-degenerate bands
- Two different momenta for backscattering:

$$q_F = |E_F| / v < k_F = |\vec{K}|$$

- Low-energy theory: restrict to these 2 bands, but include (long-ranged) Coulomb interactions

Egger & Gogolin, PRL 1997, EPJB 1998
Kane, Balents & Fisher, PRL 1997

Bosonized form

Four bosonic fields, index $a = c+, c-, s+, s-$

Low-energy theory: Luttinger liquid

$$H = \sum_a \frac{v_a}{2} \int dx \left[g_a \Pi_a^2 + g_a^{-1} (\partial_x \varphi_a)^2 \right]$$

$$g_{a \neq c+} \cong 1 \qquad g \equiv g_{c+} \approx 0.2$$

$$v_{c+} = v / g, \; v_{a \neq c+} = v$$

exactly solvable Gaussian model, leads to spin-charge separation and quasi-particles with fractional charge & fractional statistics

Experimental evidence for Luttinger liquid in SWNTs

- Tunneling density of states (many groups)
- Resonant tunneling *Postma et al., Science 2001*
- Transport in crossed geometry (no tunneling)

  Gao, Komnik, Egger, Glattli & Bachtold, PRL 2004
- Photoemission spectra (spectral function)

  Ishii, Kataura et al., Nature 2003
- STM probes of density pattern *Lee et al. PRL 2004*
- Spin-charge separation & fractionalization so far not observed in nanotubes!

Beyond lowest-order k•p scheme?

Dirac cone approximation: chirality drops out

To go beyond, one must include

- Trigonal warping: anisotropic & nonlinear dispersion relation
- Transverse momentum quantization: in parallel magnetic field *B*, including tube curvature

$$k_{y\alpha} = eBR^2 / 2h + \alpha (a / R) \cos 3\theta$$

- Net effect: R/L movers have different velocity

$$\delta = \frac{v_R - v_L}{v_R + v_L} = \frac{B}{B_0} \sin 6\theta$$

$$B_0 \sim q_F R$$

Nonlinear current-voltage relation

- Linear transport: Onsager-Casimir relation

$$G(B) = G(-B)$$

- Out of equilibrium: odd-in-B part allowed

$$I_e(V, B) = -I_e(V, -B)$$

this contribution is even in voltage!

- Fundamentally interesting because nonzero effect requires combined presence of
  - Electron-electron interactions
  - Chirality (handedness): broken inversion symmetry
  - Magnetic field: broken time reversal symmetry

Sanchez & Büttiker, PRL 2004
Spivak & Zyuzin, PRL 2004

How to include in low energy theory?

- Luttinger liquid theory now comes with chiral plasmon velocities, but still exactly solvable Gaussian theory

$$v_{c+,R/L}/v = g^{-1} \pm \delta$$

$$v_{a\neq c+,R/L} = v_{R/L} = v(1 \pm \delta)$$

- Consider long SWNT & good contacts
  - Effect requires (at least two) impurities
  - Here: 2 impurities separated by distance *d*
  - Perturbation theory in impurity strength
  - Nonequilibrium Keldysh approach

Odd-in-B current in a chiral SWNT

De Martino, Egger & Tsvelik, cond-mat/0605645

Analytical result (single-channel version):

$$I_e \propto \sin(2q_F d)\Theta^{2g-1}e^{-g\Theta}\sin\left(\frac{(1-g^2)B}{gB_0}\sin(6\theta)U\right)$$

$$\times \mathrm{Im}\left[e^{iU}\frac{\Gamma(1+g-iU/\Theta)}{\Gamma(g)\Gamma(2-iU/\Theta)}F\left(g,1+g-iU/\Theta;2-iU/\Theta;e^{-2\Theta}\right)\right]$$

with dimensionless temperature/voltage $\Theta = \dfrac{2\pi k_B T}{\hbar v/gd}, \quad U = \dfrac{|eV|}{\hbar v/gd}$

Requires interactions (g<1) and chirality (sin6θ≠0)

odd in magnetic field *B,* even in bias voltage *V*

changes sign with handedness (enantioselective)

Available experimental results

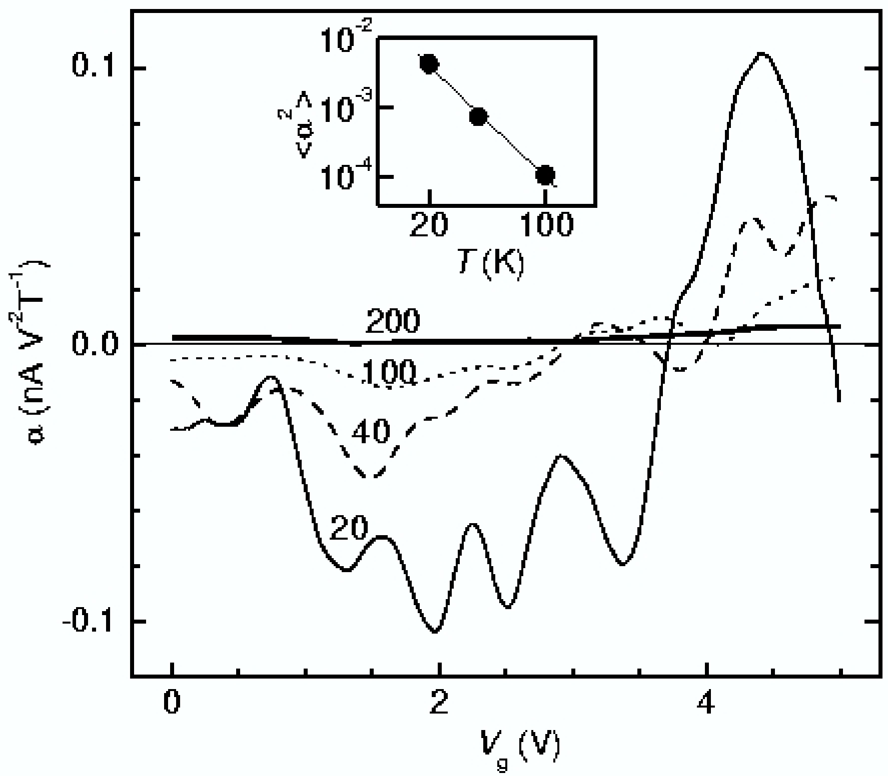
Wei, Cobden et al., PRL 2005

Measured:

$$\alpha(T) = \left[\frac{I_e(V,T,B)}{V^2 B} \right]_{V,B\to 0}$$

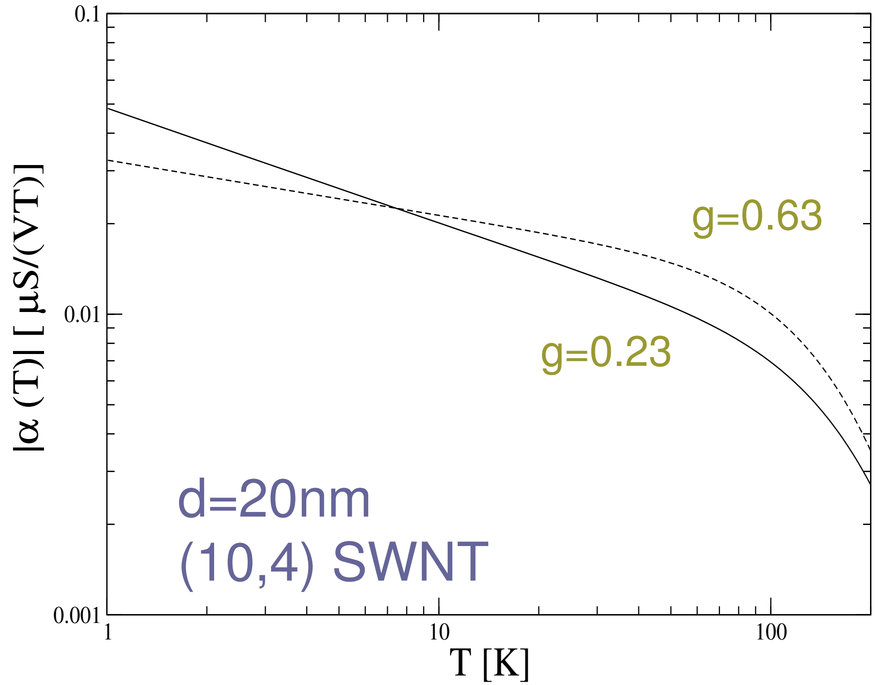
for individual SWNT (with several impurities)

- Oscillatory dependence on gate voltage; corresponds to $\sin(2q_F d)$ factor
- increases when lowering temperature



Theoretical result for *α(T)*

- Power-law scaling at low temperature:
  $$\alpha(T) \propto T^{(g-1)/2}$$
- Exponentially small at high temperature
- Order of magnitude as in experimental data
- Doesn´t change sign as function of temperature



Oscillations in $I_e(V)$

Zero temperature limit of single-channel result:

$$I_e \propto \sin\left[\frac{(1-g^2)B}{gB_0}\sin(6\theta)U\right]U^{g-1/2}J_{g-1/2}(U)$$

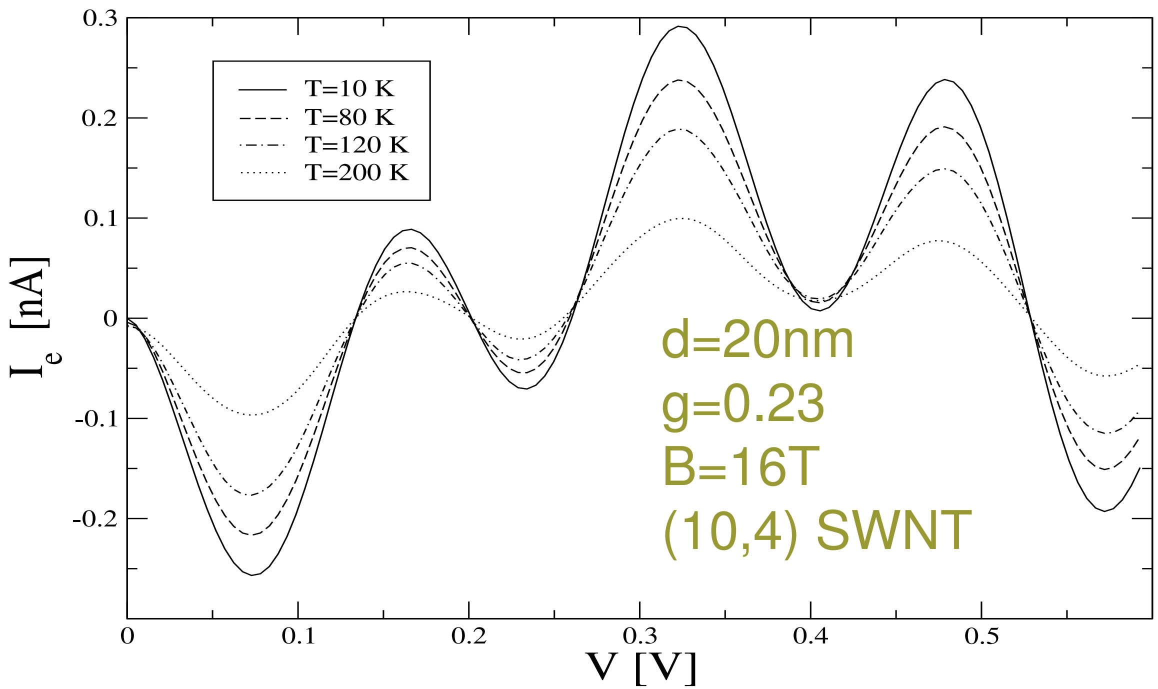
predicts oscillations as function of V with periods:

$$\Delta V_1 = \frac{hv}{egd}$$ yields Luttinger parameter

$$\Delta V_2 = \frac{B_0 g \Delta V_1}{B(1-g^2)\sin(6\theta)}$$ yields chirality

Low-voltage limit: Power-law scaling $I_e(V \to 0) \propto |V|^{2g}$

To be observed experimentally…



direct observation of interaction physics possible

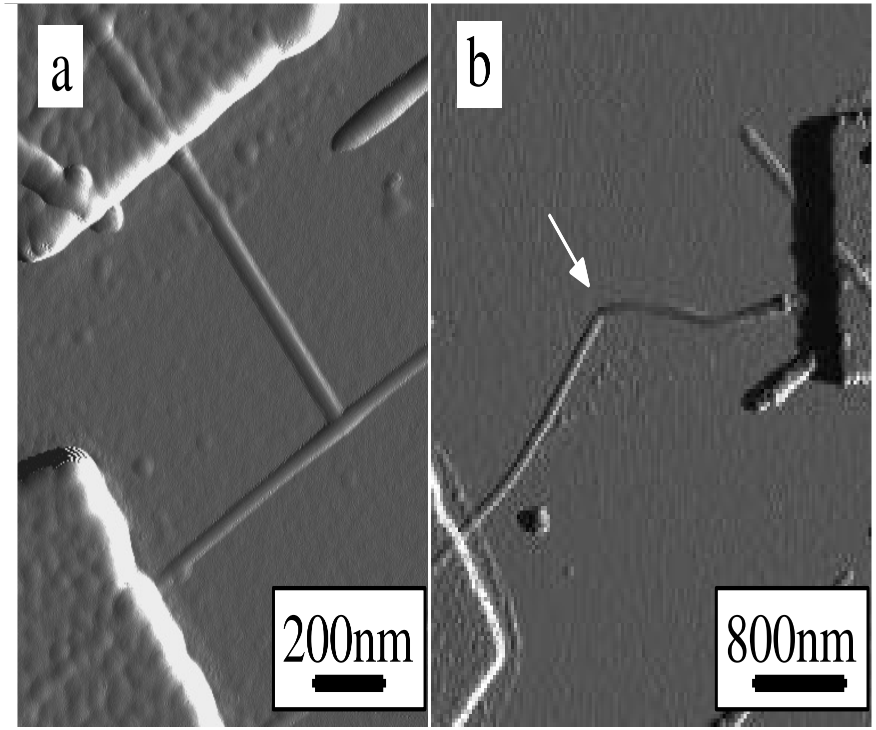
What about MWNTs?

- Electronic transport in MWNTs usually in outermost shell only
- Energy scales one order smaller
- Typically $N \approx 10$ bands due to doping
- Inner shells can also create `disorder´
  - Experiments indicate mean free path $\ell > R$
  - Ballistic behavior for $\omega\tau > 1, \tau = \ell / v$
- Also relevant for long SWNTs

Experiment: TDoS of MWNT

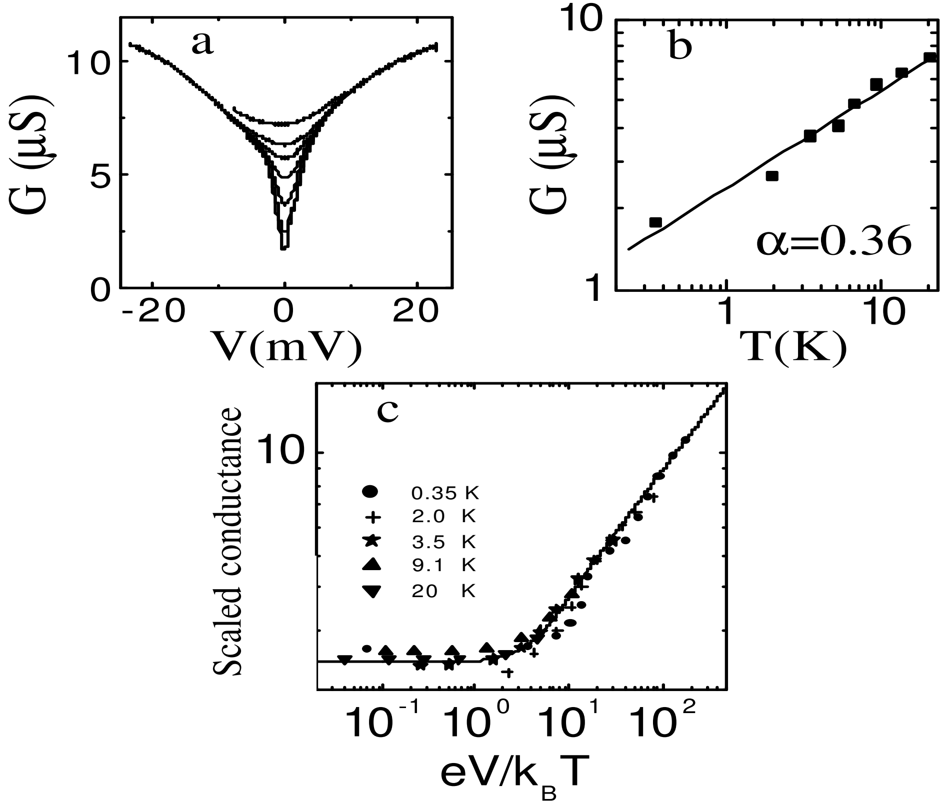
Bachtold et al., PRL 2001

- TDoS observed from conductance through tunnel contact
- Power law zero-bias anomalies
- Scaling properties similar to a Luttinger liquid, but: exponent larger than expected from Luttinger theory

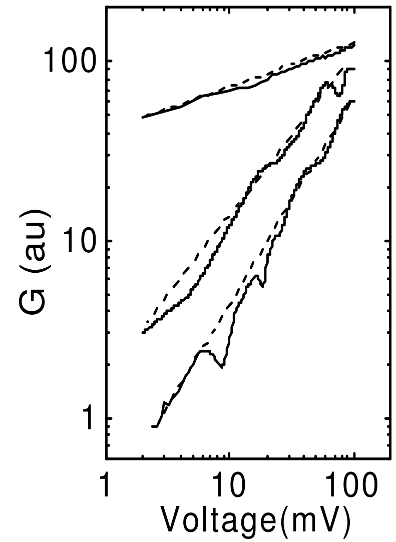


Tunneling DoS of MWNTs: experiment

Bachtold et al., PRL 2001



Geometry dependence



$$\eta_{end} = 2\eta_{bulk}$$

TDoS of multi-band Luttinger liquid

Power-law suppressed TDoS reflects orthogonality catastrophe: Electron splinters into true quasiparticles

Geometry dependence

Matveev & Glazman, PRL 1993
Egger, PRL 1999

$$\rho(x,\omega) = \mathrm{Re}\int_0^\infty dt e^{i\omega t} \left\langle \Psi(x,t)\Psi^+(x,0) \right\rangle \propto \omega^\eta$$

$$\eta_{bulk} \equiv \eta = \left(g + 1/g - 2\right)/2N$$

$$\eta_{end} = (1/g - 1)/N > 2\eta$$

Exponents are one order of magnitude too small to explain Basel experiment.

Role of disorder?

Interplay of disorder and interaction

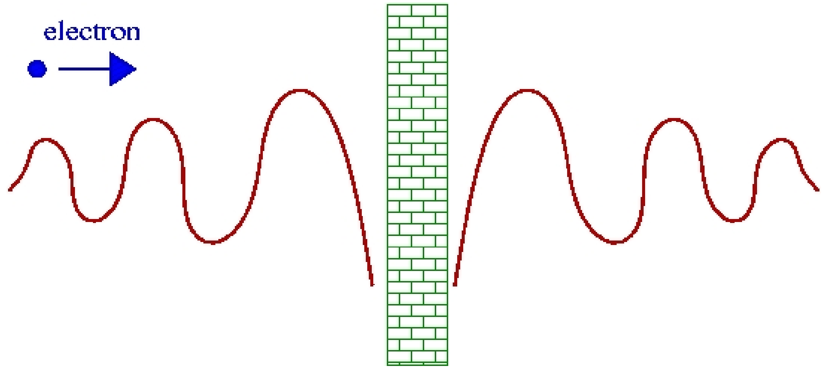
Mora, Egger & Altland, cond-mat/0602411

- Coulomb interaction enhanced by disorder
- Expected: crossover from quasiballistic Luttinger liquid at $\omega\tau > 1$ to diffusive/localized phase (e.g. Altshuler-Aronov diffusive anomalies) at $\omega\tau < 1$
- Field theory for multi-channel case and arbitrary disorder strength:

Interacting Nonlinear σ Model

Earlier versions:
Finkel'stein, Z. Phys. B 1983
Kamenev & Andreev, PRB 1999

Friedel oscillation



Mechanism unifying Luttinger liquid and Altshuler-Aronov corrections: *Matveev, Yue & Glazman, PRL 1993*

- Barrier (impurity) generates Friedel oscillation
- Incoming electron is also backscattered by Hartree-Fock potential of Friedel oscillation
- Energy dependence linked to Friedel oscillation asymptotics: very slow decay, $\delta\rho(x) \propto x^{-g}$

Egger & Grabert, PRL 1995

- Quantitative treatment of disorder difficult using this picture. Better suited: Nonlinear sigma model

Interacting nonlinear sigma model

- N-channel (N large) Luttinger liquid & weak disorder
  - Disorder forward scattering can be gauged away; backscattering: Gaussian short ranged random potential
- Replica formalism
  - Disorder average: time-nonlocal four-fermion interactions
  - Hubbard-Stratonovich field $\varphi(x,t)$ decouples electron-electron interactions
  - Matrix fields $Q_{R/L}(x,t,t')$ decouple time-nonlocal interactions
- Then saddle-point plus fluctuations scheme

Bulk TDoS

- Analytical result for $\rho(\omega, T)$ available
- Can be recast in terms of standard *P(E)* Coulomb blockade theory (microscopic derivation)

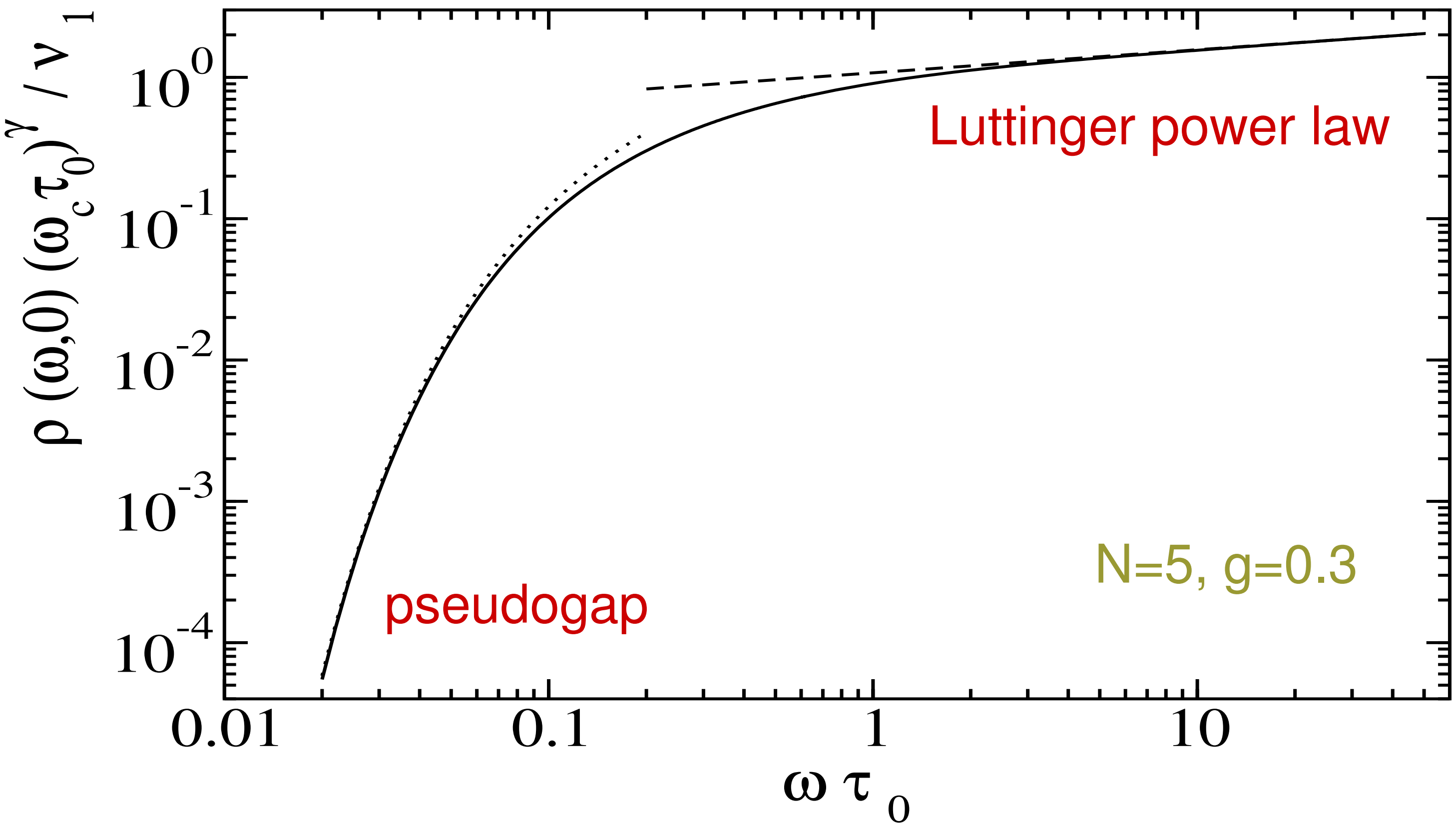
Egger & Gogolin, PRL 2001
Rollbühler & Grabert, PRL 2001

- Zero temperature: describes crossover from
  - Luttinger power law $\rho(\omega\tau > 1) \propto \omega^{\eta}$
  - to pseudogap at low energy:

$$\rho(\omega\tau < 1) \propto \frac{\sqrt{\omega\tau}}{\eta_{end}} \exp\left(-\frac{2\pi\eta_{end}^2}{\omega\tau} \right)$$

Nazarov, JETP 1989
Mishchenko et al., 2001

Bulk TDoS at T=0



Stronger suppression of TDoS due to disorder.
But does not really explain experimental results…

Interaction correction to conductivity

Complete crossover solution from ballistic to diffusive case, to lowest order in interaction:

$$\frac{\sigma(T)}{\sigma_{Drude}} = 1 + \gamma \ln(T\tau') - \gamma \int_0^\infty d\Omega \frac{\partial_\Omega \left[\Omega \coth \frac{\hbar\Omega}{2k_B T}\right]}{\Omega}$$

$$\times \left((1 + i/\Omega\tau')^{-1/2} - 1 + \frac{i}{2\tau'(1+g)\sqrt{\Omega^2 + i\Omega/\tau'}} \right)$$

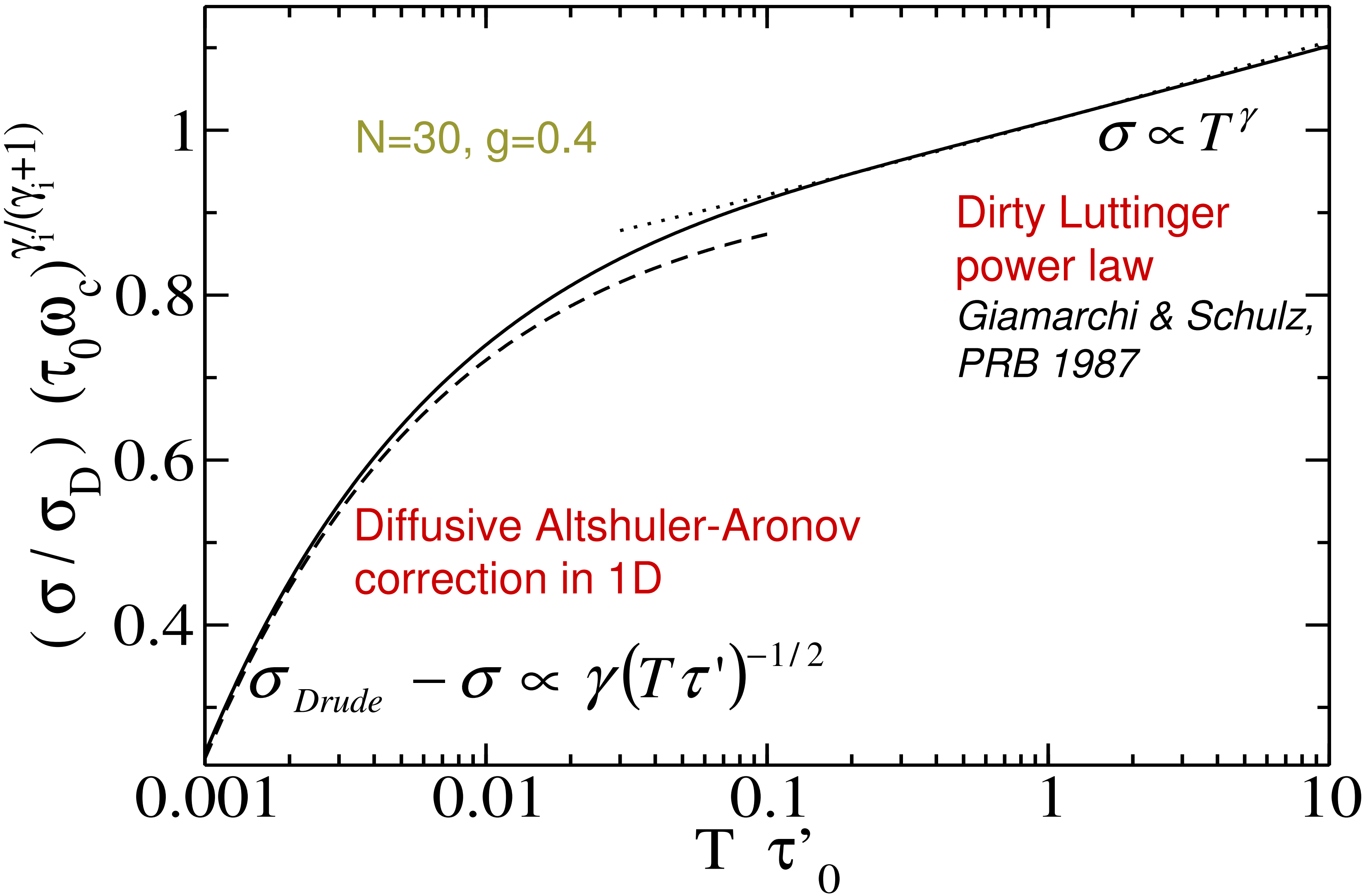
Exponent for weak backscattering by single impurity in a Luttinger liquid

$$\gamma = 2(1-g)/N$$

Renormalized mean free time

$$\tau' = \tau(\omega_c \tau)^{-\gamma/\gamma+1}$$

Conductivity



Conclusions

- Magnetotransport: Linear in B terms in the current?
  - Only present when out of equilibrium (current is even in voltage) and with (at least two) impurities
  - Only present with interactions and chirality
  - Prediction: Oscillations with bias voltage, power law scaling in T dependence,…

De Martino, Egger & Tsvelik, cond-mat/0605645

- Crossover from ballistic to diffusive regime
  - Appears (e.g.) in MWNTs
  - Large *N* allows for nonlinear sigma model description
  - Tunneling density of states
  - Conductivity: Interaction corrections

Mora, Egger & Altland, cond-mat/0602411