Magnetic barriers in graphene

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Overview

Ref.: De Martino, Dell'Anna & Egger, PRL 98, 066802 (2007)

- Introduction to graphene
- Dirac-Weyl equation
  - Effects of disorder and interactions
  - Klein paradoxon
  - Inhomogeneous magnetic fields
  - (integer) Quantum Hall Effect
- Magnetic barrier
- Magnetic quantum dot

not discussed in this talk: superconductivity in graphene, bi- or multilayer, phonon effects etc.

Graphene

review article: Geim & Novoselov, Nat. Mat. 6, 183 (2007)

- Graphene monolayers: prepared by mechanical exfoliation in 2004 & by epitaxial growth in 2005 (but different properties!)

  Novoselov et al., Science 2004, Nature 2005,
  Zhang et al. Nature 2005, Berger et al., Science 2006

- „Parent system" of many carbon-based materials (nanotubes, fullerene, graphite)
- Tremendous research activity at present

Graphene

- Monolayer graphene sheets (linear dimension of order 1*mm*) have been fabricated
  - on top of non-crystalline substrates
  - suspended membrane
  - in liquid suspension
- Technologically interesting: high mobility (comparable to good Si MOSFET), even at room temperature

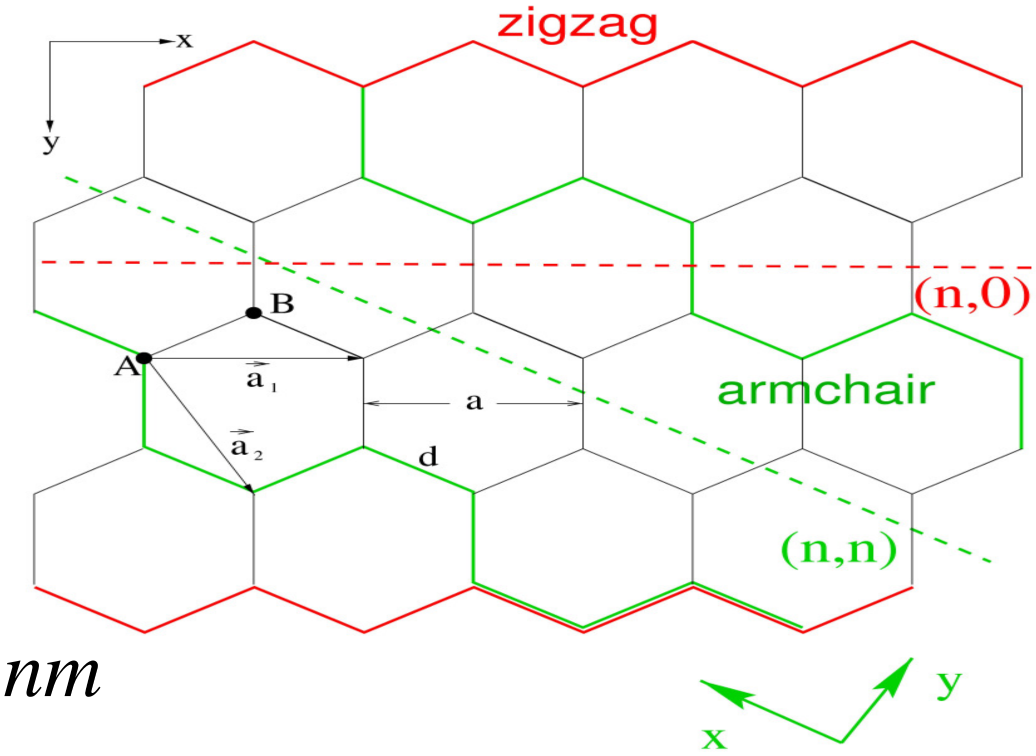
Graphene: a new 2DEG

- 2DEG represents surface state: possibility to probe by STM/AFM/STS techniques
- Electron-phonon coupling: spontaneous „crumpling“ of suspended monolayer reflects instability of 2D membrane *Meyer et al., Nature 2007*
- Electronic transport
  - „Half-integer“ Quantum Hall effect
  - „Universal conductivity“ (undoped limit)
  - Perfect (Klein) tunneling through barriers
  - Aspects related to Dirac fermion physics

Graphene: Tight binding description

Wallace, Phys. Rev. 1947

Basis contains two atoms; nearest-neighbor hopping connects different sublattices

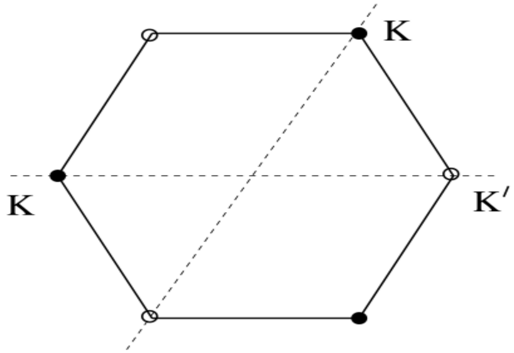


$$a = \sqrt{3}d, d = 0.14nm$$

Band structure

Exactly two independent corner points K, K´ in first Brillouin zone.

Band structure: valence and conduction bands touch at corner points *(E=0)*, these are the Fermi points in undoped graphene



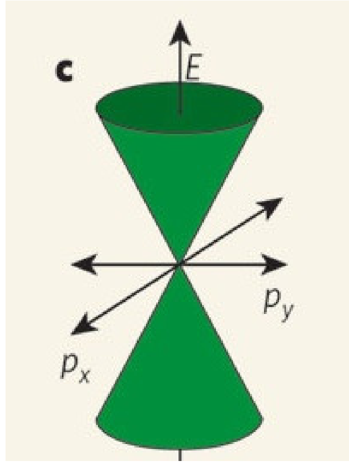
- Low energies: Dirac light cone dispersion
- Deviations at higher energies: trigonal warping

$$E(\vec{q}) = \pm\hbar v|\vec{q}|$$

$$\vec{q} = \vec{k} - \vec{K}$$

$$v \approx 10^6 m/\sec$$

Dirac Weyl Hamiltonian



Low energy continuum limit:
massless relativistic quasiparticles

$$H = H_K + H_{K'} = v\int d^2r\ \Psi^+(-i\hbar\nabla\cdot\vec{\sigma})\Psi$$

8 component spinor quantum field:
spin, sublattice, K point („valley“) degeneracy

$$\Psi(x,y) = (\Psi_{K,\uparrow,A}, \Psi_{K,\uparrow,B}, \cdots, \Psi_{K',\downarrow,B})$$

Pauli matrices in sublattice space: $\vec{\sigma} = (\sigma_x, \sigma_y)$

Electron-electron interactions

- Kinetic and Coulomb energy both scale linearly in density ↷ interaction parameter r_s not tunable by gate voltage
- Simple estimate: $r_s \approx 1$
  - RG theory: interactions scale to weak coupling
  - Fermi liquid theory holds, but not RPA

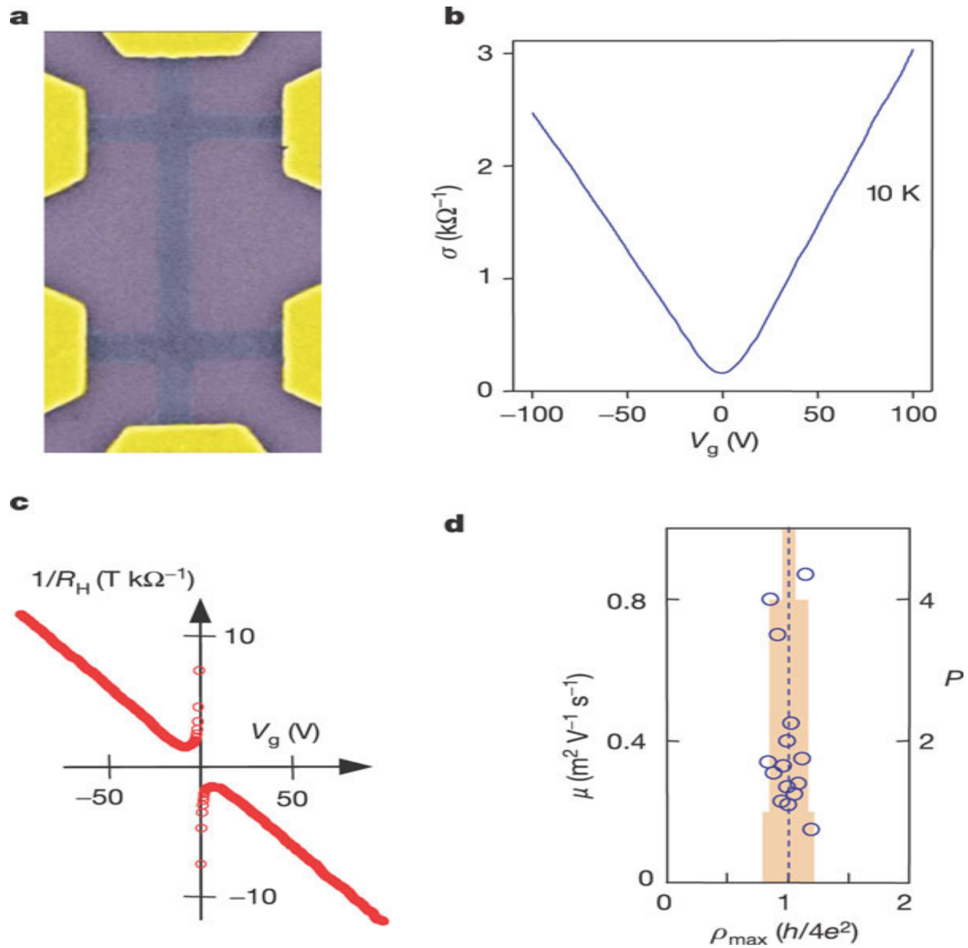
    Mishchenko, PRL 2007
  - Experiments observe near cancellation of exchange and correlation energy *Martin et al., cond-mat/0705.2180*
- no spectacular deviations from noninteracting predictions expected
  - Exceptions exist, e.g., asymmetric-in-*B* part of *IV* curve

    De Martino, Egger & Tsvelik, PRL 2006
- In the following: disregard electron-electron interaction

Disorder effects

Two experimental puzzles

- Universal minimum conductivity ~$4e^2/h$
- Linear dependence of conductivity on doping



Novoselov et al., Nature 2005

Theoretical implications

Experimental data can be rationalized only if short-range impurity scattering suppressed

- Dominant mechanism: long-ranged Coulomb scattering by defects *Nomura & MacDonald, PRL 2007*
- Then no K-K´ mixing
- Otherwise: strong localization expected *Altland, PRL 2006*
- Universal „minimum conductivity“ currently subject to considerable & hot theoretical debate

Badarzon, Twordzydlo, Brouwer & Beenakker, cond-mat/0705.0886, Ostrovsky, Gornyi & Mirlin, PRB 2006

Universal minimum conductivity?

Subtle issue…

compare order of limits for the optical conductivity of clean system at low frequency

$$\lim_{\omega\to 0}\sigma(\omega,\ell=\infty)=\frac{\pi}{8}\frac{4e^2}{h}$$

Ludwig et al., PRB 1994

$$\lim_{\ell\to\infty}\sigma(\omega=0,\ell)=\frac{1}{\pi}\frac{4e^2}{h}$$

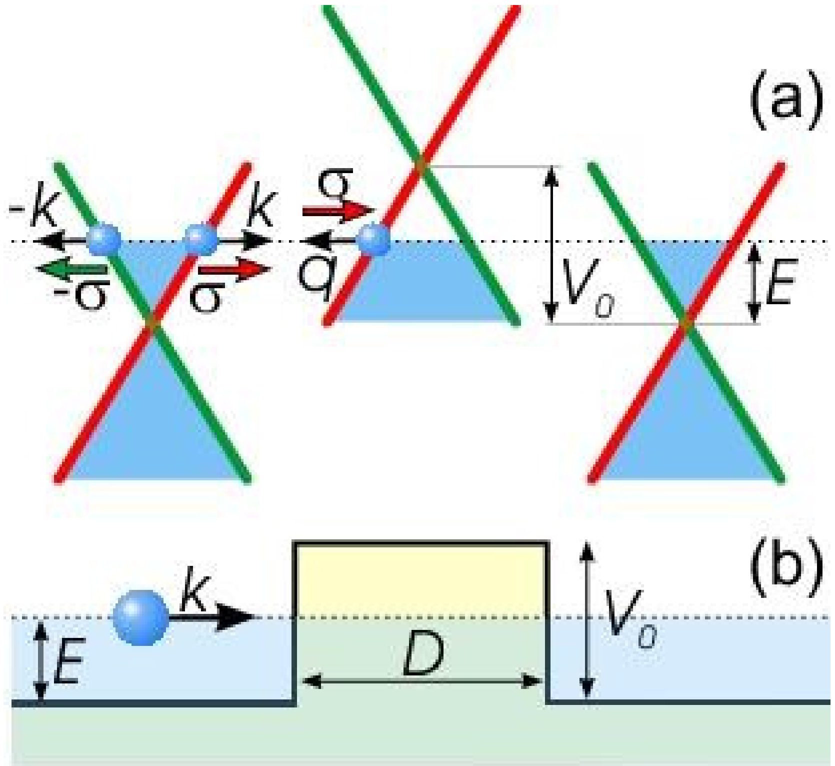
Disorder would have to increase conductivity to explain experimental data…

Klein tunneling

O.Klein, Z. Phys. B 1929
Katsnelson et al, Nature Phys. 2006

- Dirac fermions can perfectly tunnel through high and wide barrier
  - Electron and hole encoded in same equation (spinor!):

  Charge-Conjugation Symmetry
- Graphene provides good opportunity to study this effect *Williams, Di Carlo & Marcus, cond-mat 0704.3487*
- But: Confinement by electrostatic fields (gates) is then difficult



Electrostatic confinement

- Smooth electrostatic potentials:
  K-K´ scattering suppressed
- Single K point theory: Klein tunneling most pronounced for normal incidence on barrier, other states may be reflected

Silvestrov & Efetov, PRL 2007

- How to produce mesoscopic structures? (quantum point contacts, quantum wires, quantum dots etc.)
- Our proposal: use magnetic barriers

Inhomogeneous magnetic field

Perpendicular orbital magnetic field

$$\vec{B} = B(x,y)\vec{e}_z = \nabla \times \vec{A}$$

- Simplest level: ignore Zeeman field (and e-e interaction) → electron spin irrelevant
- Consider ballistic case (for simplicity)
  - Disorder mostly of long-range type, preserves valley degeneracy *Nomura & MacDonald, PRL 2006*
- For smooth field variation (on scale *a*):

  K and K´ states remain decoupled, focus on single K point theory

Now: „minimal substitution“ $-i\hbar\nabla \rightarrow -i\hbar\nabla + e\vec{A}$

Dirac-Weyl equation with magnetic field

$$\left(-i\hbar\nabla + e\vec{A}\right)\cdot\vec{\sigma}\begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix} = \varepsilon\begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix}$$

equivalent to pair of decoupled Schrödinger-like equations:

$$\left(\left(-i\hbar\nabla + e\vec{A}\right)^2 + e\sigma_z B_z - \varepsilon^2\right)\Psi = 0$$

- Energies come in plus-minus pairs (chiral Hamiltonian)
- Zeeman-like term in sublattice space

Homogeneous field

$$B(x, y) = B_0$$

Relativistic Landau levels, 4-fold degenerate

$$E_n = \operatorname{sgn}(n) v \sqrt{2 e B_0 |n|}$$

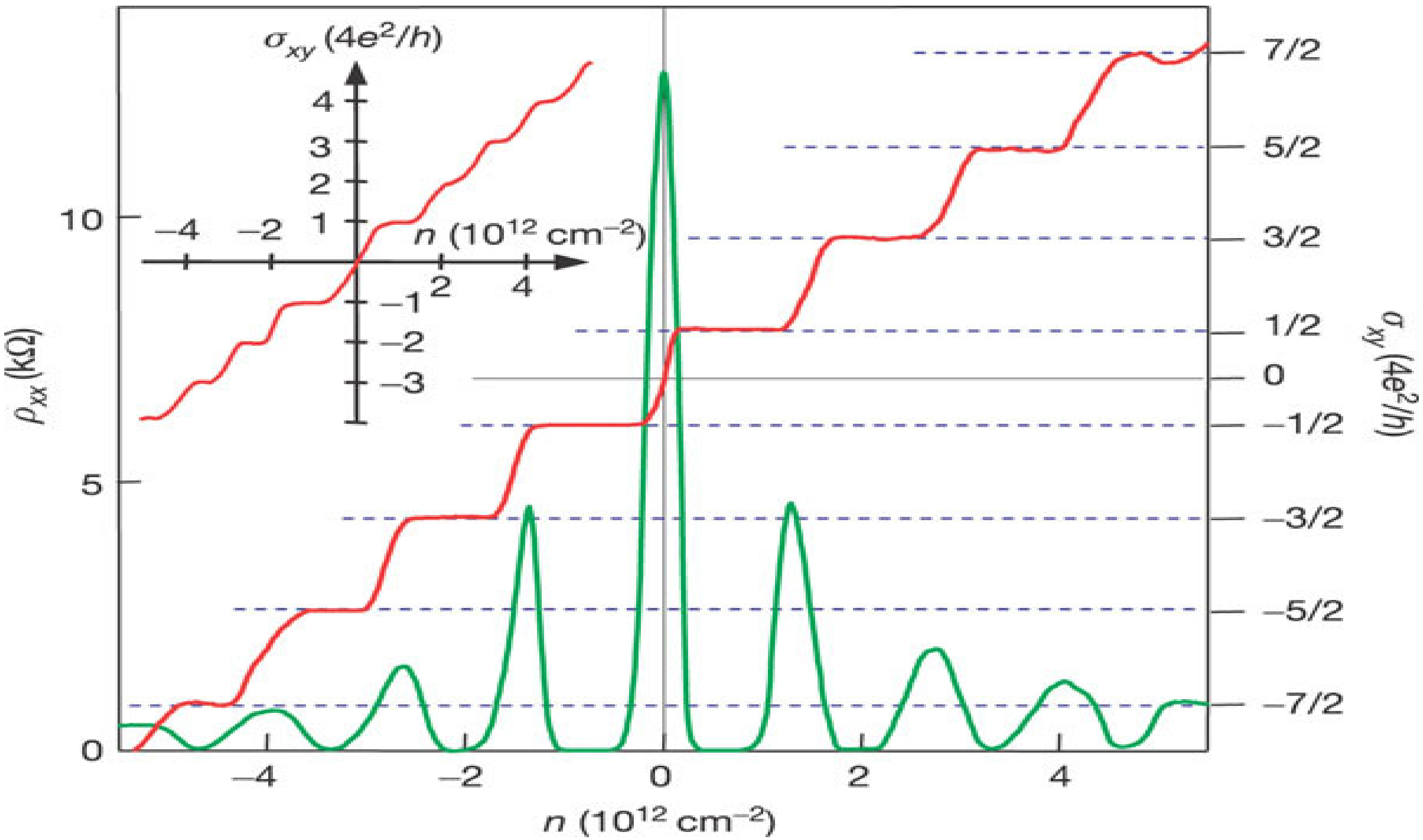
results in „half-integer“ QHE because of presence of zero-energy state

$$\sigma_{xy} = \frac{4e^2}{h}\left(n + \tfrac{1}{2}\right)$$

Experimentally confirmed

Zhang et al., Nature 2005, Novoselov et al., Nature Phys. 2006

Integer QHE in graphene: expt. data



Magnetic barrier: Model

Consider square barrier:

$$B(x,y) = \begin{cases} B_0, & |x| < d \\ 0, & |x| > d \end{cases}$$

Good approximation for

$$\lambda_F > \lambda_B > a$$

↑ edge smearing length

Convenient gauge:

$$\vec{A} = B_0 \vec{e}_y \cdot \begin{cases} -d, & x < -d \\ x, & |x| < d \\ d, & x > d \end{cases}$$

y component of momentum conserved!

Magnetic barrier: Solution

… pair of decoupled 1D Schrödinger eqns (assume electron-like state $\varepsilon > 0$)

$$\left(-\partial_x^2 + V_{A/B}(x) - \varepsilon^2\right)\psi_{A/B}(x) = 0$$

Effective potentials $V_{A/B}(x) = \pm eA_y(x) + \left(p_y + eA_y(x)\right)^2$

parametrize momentum by kinematic incidence angle

$$k_x = \varepsilon \cos \phi$$

$$k_y = \frac{p_y}{\hbar} = \varepsilon \sin \phi + edB_0$$

Gauge invariant velocity: $\vec{v} = v\begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix}$

Incoming scattering state (from left)

Left of the barrier: $\Psi_{left} = e^{ik_x x}\begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix} + re^{-ik_x x}\begin{pmatrix} 1 \\ -e^{-i\phi} \end{pmatrix}$

Under the barrier:

$$l_B = \sqrt{\hbar / eB_0}$$

$$\Psi_{barrier} = \sum_{\pm} c_{\pm}\begin{pmatrix} D_{-1+(\varepsilon l_B)^2/2}\left(\pm\sqrt{2}\left(k_y l_B + x/l_B\right)\right) \\ \pm i\dfrac{\sqrt{2}}{\varepsilon l_B} D_{(\varepsilon l_B)^2/2}\left(\pm\sqrt{2}\left(k_y l_B + x/l_B\right)\right) \end{pmatrix}$$

Right of the barrier: $\Psi_{right} = t\sqrt{k_x / k'_x}\; e^{ik'_x x}\begin{pmatrix} 1 \\ e^{i\phi'} \end{pmatrix}$

with emergence angle in $k'_x = \varepsilon\cos\phi'$

Perfect reflection regime

- Transmission/reflection probability

$$T = |t|^2, R = |r|^2 = 1 - T$$

- Relation between emergence and incidence angle from y-momentum conservation

$$\sin\phi' - \sin\phi = 2d / \varepsilon l_B^2$$

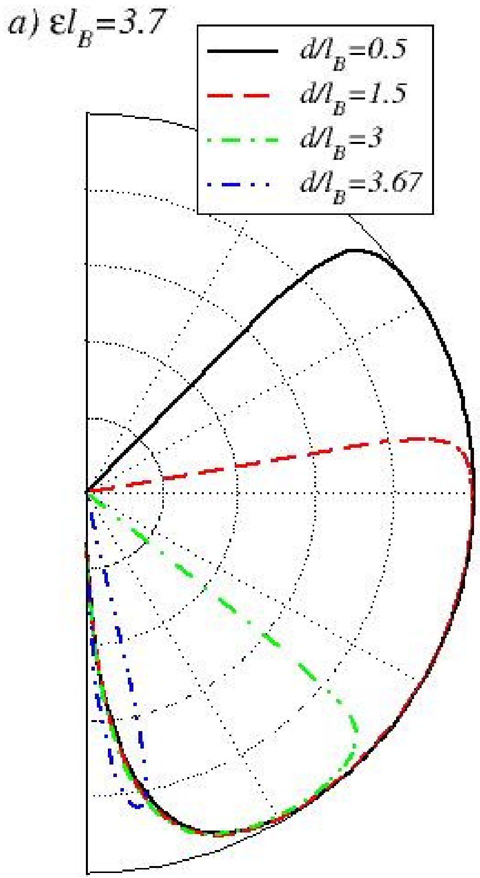
- No solution, i.e. perfect reflection, for low energy and/or wide barrier

$$\varepsilon l_B < d / l_B$$

opens up possibility of confining Dirac Weyl quasiparticles

Transmission probability

angular plot of transmission probability $T(\phi)$ (away from the perfect reflection regime)



Magnetic quantum dot

- Circularly symmetric magnetic field $\vec{B} = B(r)\vec{e}_z$
- Total angular momentum $J = -i\partial_\theta + \sigma_z/2$ is conserved, good quantum number $j = m \pm 1/2$
- gives Dirac-Weyl radial (1D) equations

$$\begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \begin{pmatrix} e^{im\theta}\phi_m(r) \\ e^{i(m+1)\theta}\chi_m(r) \end{pmatrix}$$

$$\frac{d\phi_m}{dr} - \frac{m+\varphi(r)}{r}\phi_m = i\varepsilon\chi_m$$

$$\frac{d\chi_m}{dr} + \frac{m+1+\varphi(r)}{r}\chi_m = i\varepsilon\phi_m$$

Magnetic flux through disc of radius *r* in flux quanta

$$\varphi(r) = e\int_0^r r'dr'B(r')$$

Simple model for magnetic dot

Again simple step-type model: $B(r) = \begin{cases} 0, & r < R \\ B_0, & r > R \end{cases}$

Solution:

missing flux through dot (in flux quanta)

$$\delta = R^2 / 2l_B^2$$

$$\tilde{m} = m - \delta$$

$$\xi = r^2 \big/ 2l_B^2$$

$$\phi_m(r < R) = a_< J_m(\varepsilon r)$$

$$\phi_m(r > R) = a_> \xi^{|\tilde{m}|/2} e^{-\xi/2}$$

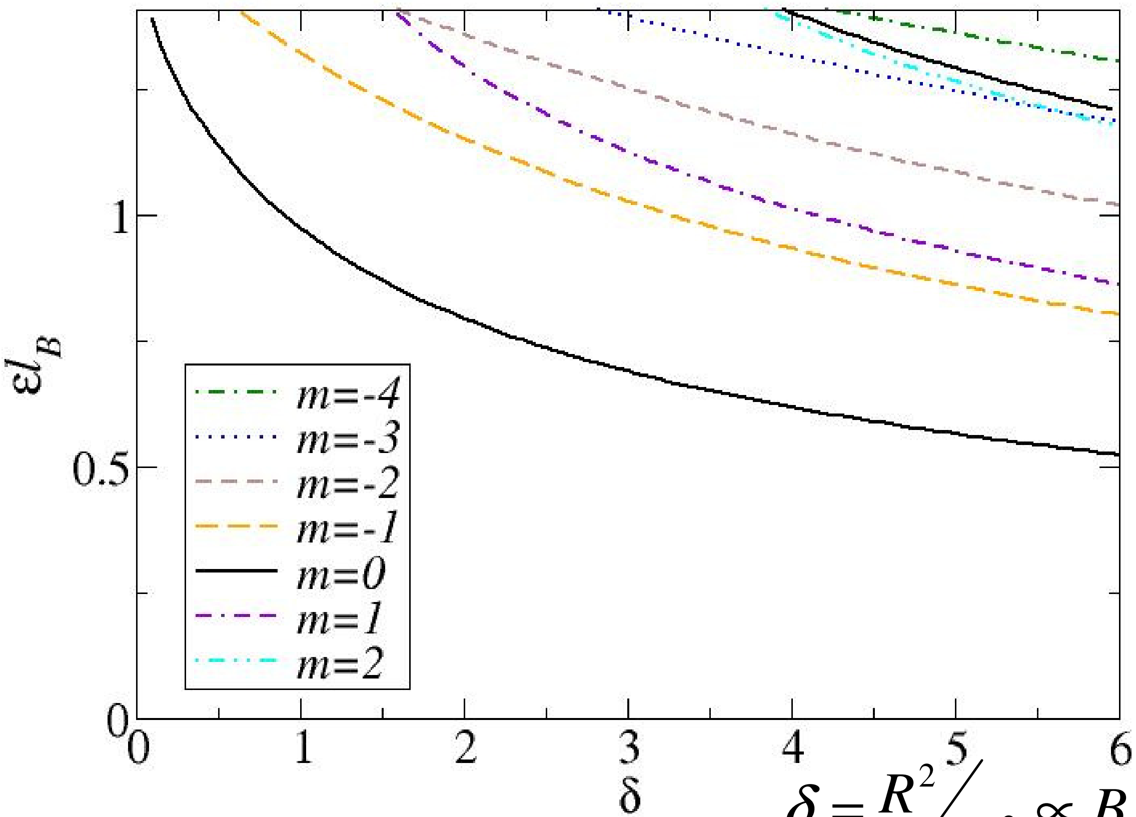
$$\times \Psi\left(1 + \tilde{m}\,\theta(\tilde{m}) - \frac{\varepsilon^2 l_B^2}{2}, 1 + |\tilde{m}|; \xi\right)$$

degenerate hyper-geometric function

Matching problem gives energy quantization condition!

Magnetic dot eigenenergies

(above zero, but below first bulk Landau level)



$\delta = R^2 / 2l_B^2 \propto B_0$

Estimate:

$B_0 = 4T \Rightarrow l_B = 13nm,$

$\varepsilon l_B = 1 \Leftrightarrow E = 44meV$

Energy levels tunable via magnetic field

Conclusions

- Graphene as model 2DEG system made of relativistic Dirac fermions
- Klein tunneling: Dirac fermions cannot be easily trapped by electrostatic fields
- Magnetic fields (inhomogeneous) can confine Dirac fermions. Solution discussed for
  - Magnetic barrier (square barrier)
  - Magnetic dot (circular confinement)