Theory of electronic transport in carbon nanotubes

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Electronic transport in nanotubes

Most mesoscopic effects have been observed

(see seminar of C.Schönenberger)

- Disorder-related: MWNTs
- Strong-interaction effects
- Kondo and dot physics
- Superconductivity
- Spin transport
- Ballistic, localized, diffusive transport

- What has theory to say?

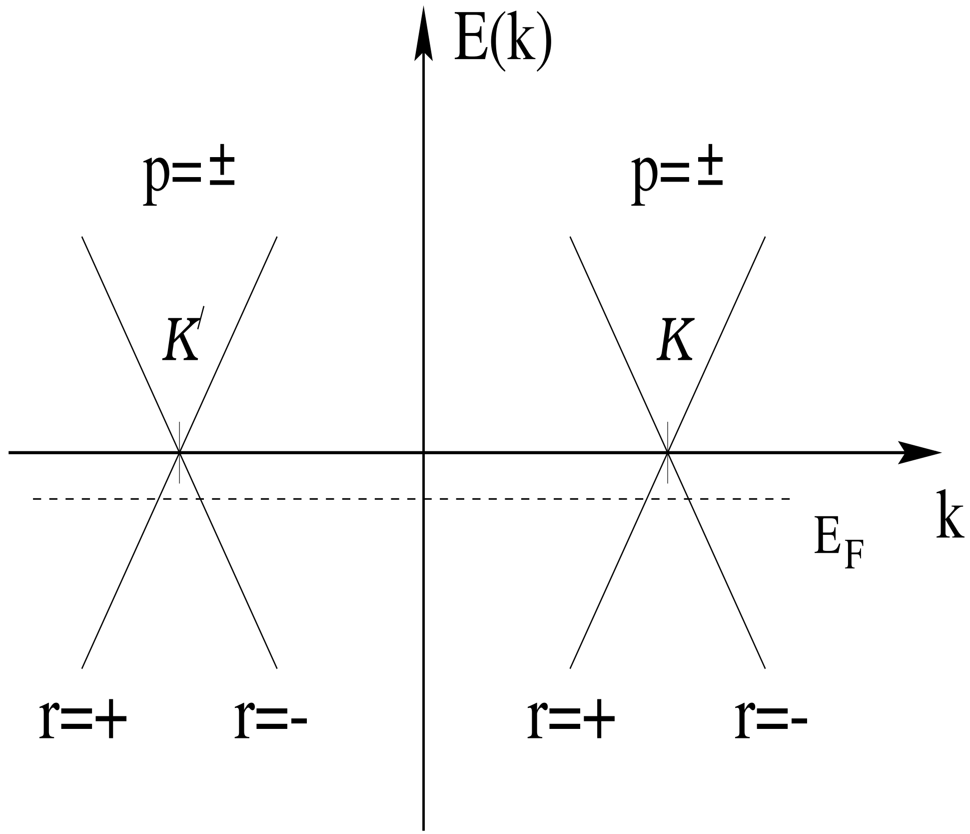
Overview

- Field theory of ballistic single-wall nanotubes: Luttinger liquid and beyond (A.O. Gogolin)
- Multi-terminal geometries
  - Y junctions (S. Chen & B. Trauzettel)
  - Crossed nanotubes: Coulomb drag (A. Komnik)
- Multi-wall nanotubes: Nonperturbative Altshuler-Aronov effects (A.O. Gogolin)
- Superconductivity in ropes of nanotubes (A. De Martino)

Metallic SWNTs: Dispersion relation

- Basis of graphite sheet contains two atoms: two sublattices *p=+/-*, equivalent to right/left movers *r=+/-*
- Two degenerate Bloch waves at each Fermi point *K,K´ (α=+/-)*

$$\phi_{p\alpha}(x, y)$$



SWNT: Ideal 1D quantum wire

- Transverse momentum quantization: $k_y = 0$ is only relevant transverse mode, all others are far away
- 1D quantum wire with two spin-degenerate transport channels (bands)
- Massless 1D Dirac Hamiltonian
- Two different momenta for backscattering:

$$q_F = |E_F| / v_F < k_F = |\vec{K}|$$

What about disorder?

- Experimentally observed mean free paths in high-quality metallic SWNTs $\ell \geq 1\mu m$
- Ballistic transport in not too long tubes
- No diffusive regime: Thouless argument gives localization length $\xi = N_{bands}\,\ell = 2\ell$
- Origin of disorder largely unknown. Probably substrate inhomogeneities, defects, bends and kinks, adsorbed atoms or molecules,…
- For now focus on ballistic regime

Field theory of interacting SWNTs

Egger & Gogolin, PRL 1997, EPJB 1998
Kane, Balents & Fisher, PRL 1997

- Keep only two bands at Fermi energy
- Low-energy expansion of electron operator:

$$\Psi_\sigma(x,y) = \sum_{p,\alpha} \psi_{p\alpha\sigma}(x)\phi_{p\alpha}(x,y)$$

$$\phi_{p\alpha}(x,y) = \frac{1}{\sqrt{2\pi R}} e^{-i\alpha\vec{K}\cdot\vec{r}}$$

- 1D fermion operators: Bosonization applies
- Inserting expansion into full SWNT Hamiltonian gives 1D field theory

Interaction potential (no gates…)

- Second-quantized interaction part:

$$H_I = \frac{1}{2}\sum_{\sigma\sigma'} \int d\vec{r}\, d\vec{r}\,' \Psi_\sigma^+(\vec{r}\,)\Psi_{\sigma'}^+(\vec{r}\,')$$
$$\times\, U\,(\vec{r} - \vec{r}\,')\Psi_{\sigma'}(\vec{r}\,')\Psi_\sigma(\vec{r}\,)$$

- Unscreened potential on tube surface

$$U = \frac{e^2/\kappa}{\sqrt{(x-x')^2 + 4R^2\sin^2\left[\frac{y-y'}{2R}\right] + a_z^2}}$$

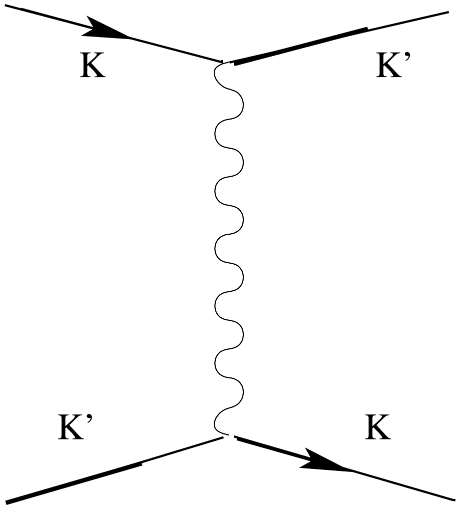
1D fermion interactions

- Insert low-energy expansion
- Momentum conservation allows only two processes away from half-filling
  - Forward scattering: „Slow“ density modes, probes long-range part of interaction
  - Backscattering: „Fast“ density modes, probes short-range properties of interaction
  - Backscattering couplings scale as *1/R*, sizeable only for ultrathin tubes

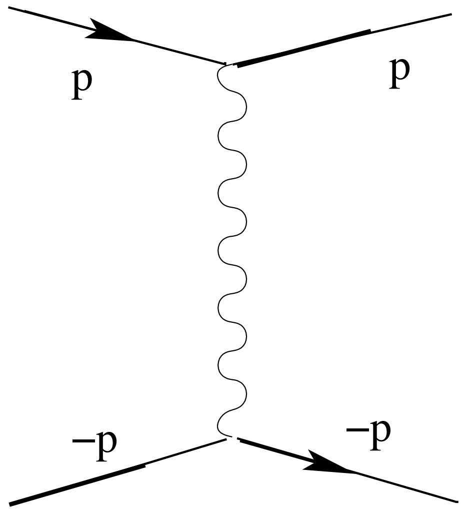
Backscattering couplings

Momentum exchange

$2k_F$



$2q_F$



with coupling constant

$$b = 0.1e^2 / R$$

$$f = 0.05e^2 / R$$

Bosonized form of field theory

- Four bosonic fields, index $a = c+, c-, s+, s-$
  - Charge (c) and spin (s)
  - Symmetric/antisymmetric K point combinations
- Luttinger liquid & nonlinear backscattering

$$H = \frac{v_F}{2} \int dx \sum_a \left[\Pi_a^2 + g_a^{-2} (\partial_x \varphi_a)^2 \right] +$$

$$+ f \int dx \left[-\cos\varphi_{c-} \cos\varphi_{s-} - \cos\varphi_{c-} \cos\varphi_{s+} + \cos\varphi_{s-} \cos\varphi_{s+} \right] +$$

$$+ b \int dx \left[\cos\varphi_{s-} + \cos\vartheta_{s-} \right] \cos\varphi_{c-}$$

Luttinger parameters for SWNTs

- Bosonization gives $g_{a \neq c+} \cong 1$
- Logarithmic divergence for unscreened interaction, cut off by tube length

$$g \equiv g_{c+} = \left[1 + \frac{8e^2}{\pi\kappa\hbar v_F} \ln\left(L/2\pi R \right) \right]^{-1/2} =$$

$$= \frac{1}{\sqrt{1 + 2E_c / \Delta}} \approx 0.2$$

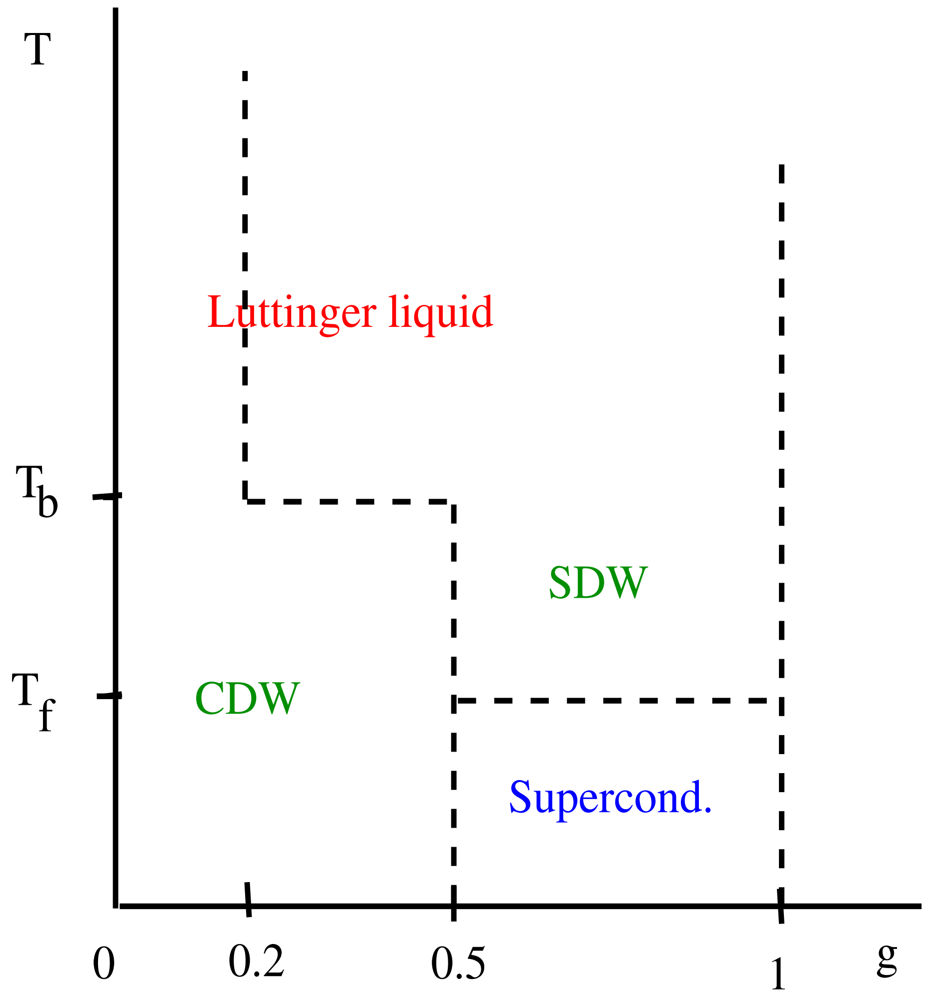
- Pronounced non-Fermi liquid correlations

Phase diagram (quasi long range order)

- Effective field theory can be solved in practically exact way
- Low temperature phases matter only for ultrathin tubes or in sub-mKelvin regime

$$T_f = (f/b)T_b$$

$$k_B T_b = D e^{-v_F/b} \propto e^{-R/R_b}$$



Tunneling DoS for nanotube

- Power-law suppression of tunneling DoS reflects orthogonality catastrophe: Electron has to decompose into true quasiparticles
- Experimental evidence for Luttinger liquid in tubes available from TDoS
- Explicit calculation gives

$$\nu(x,E) = \mathrm{Re}\int_0^\infty dt e^{iEt} \left\langle \Psi(x,t)\Psi^+(x,0) \right\rangle \propto E^\eta$$

- Geometry dependence:

$$\eta_{bulk} = \left(g + 1/g - 2\right)/4$$

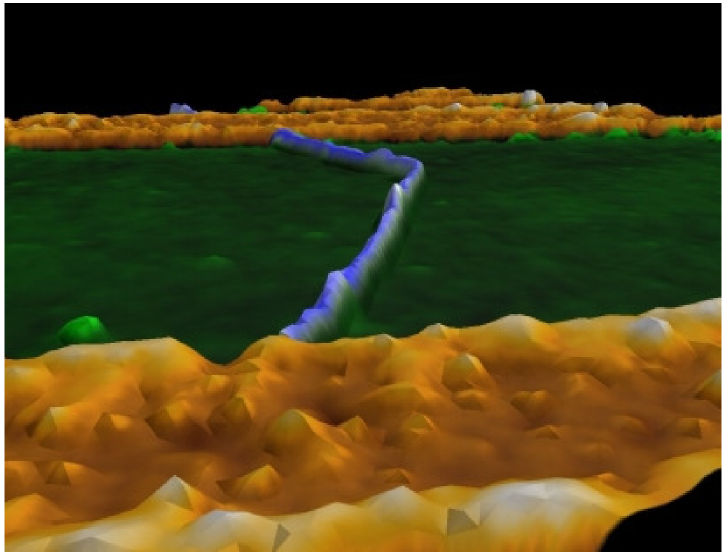
$$\eta_{end} = (1/g - 1)/2 > 2\eta_{bulk}$$

Conductance probes tunneling DoS

- Conductance across kink:

$$G \propto T^{2\eta_{end}}$$

- Universal scaling of nonlinear conductance:

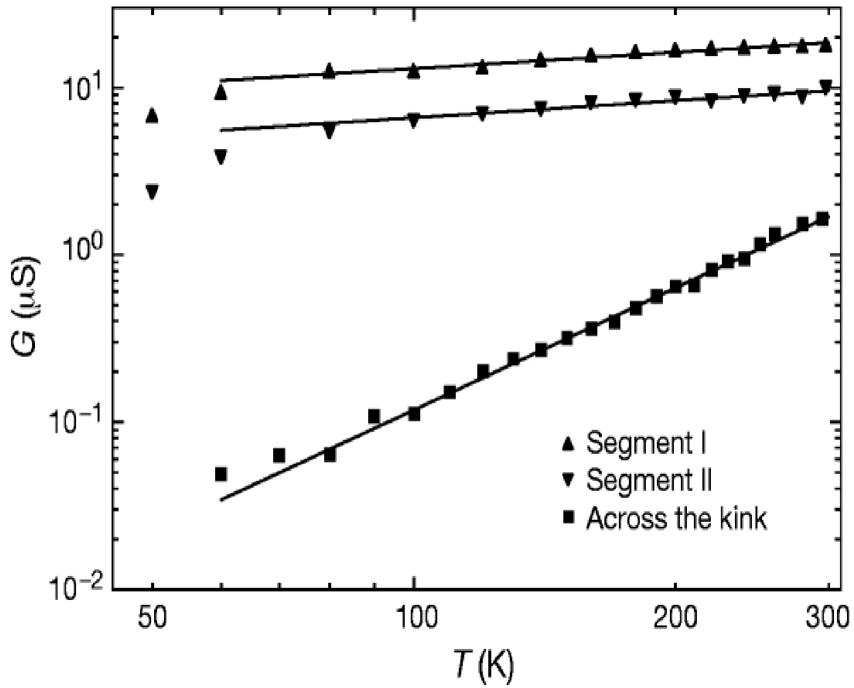


Delft group

$$T^{-2\eta_{end}} dI/dV \propto \sinh\left[\frac{eV}{2k_B T}\right]\left|\Gamma\left(1+\eta_{end}+\frac{ieV}{2\pi k_B T}\right)\right|^2$$

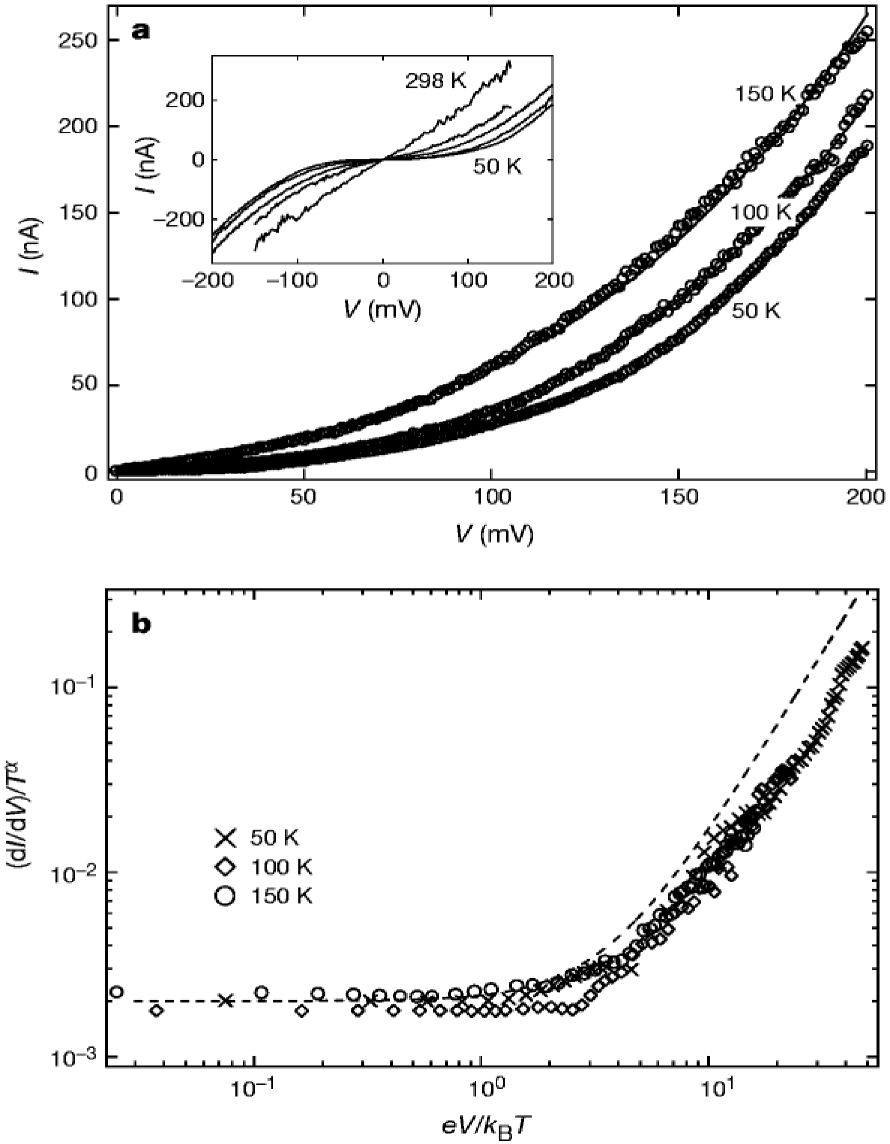
$$\cdot\left[\coth\left(\frac{eV}{2k_B T}\right)-\frac{1}{2\pi}\operatorname{Im}\Psi\left(1+\eta_{end}+\frac{ieV}{2\pi k_B T}\right)\right]$$

Evidence for Luttinger liquid



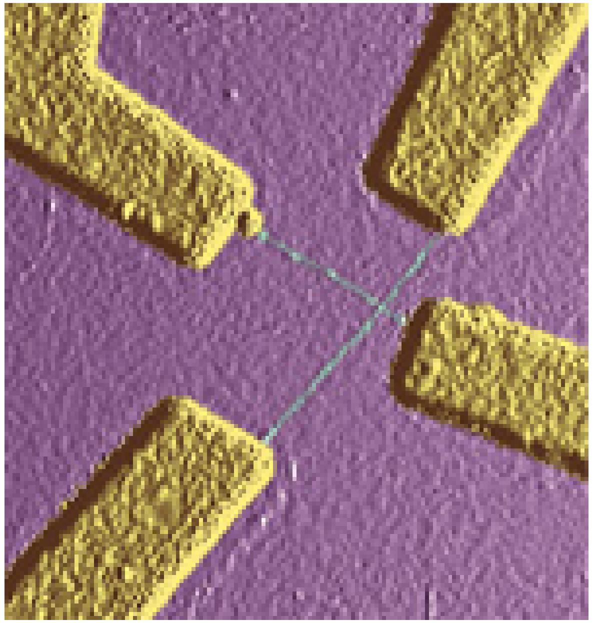
gives *g* around 0.22

Yao et al., Nature 1999



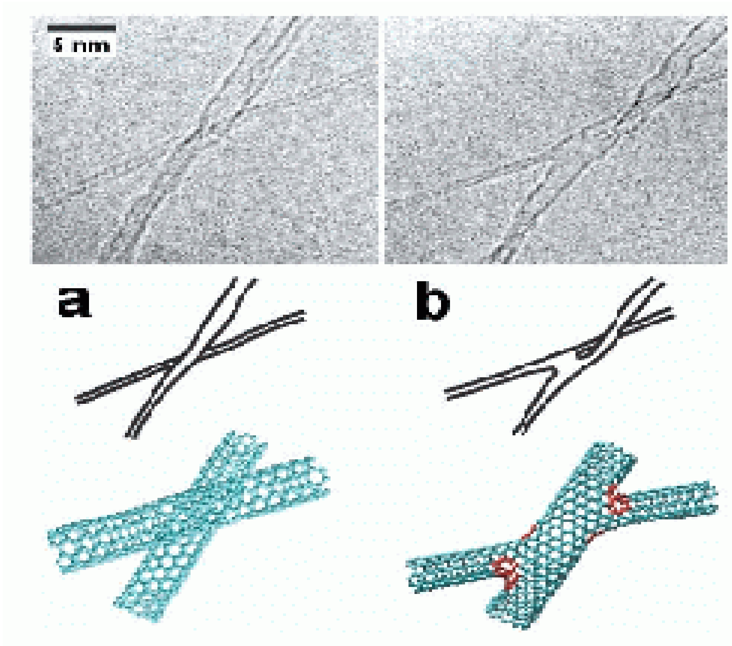
Multi-terminal circuits: Crossed tubes

By chance…



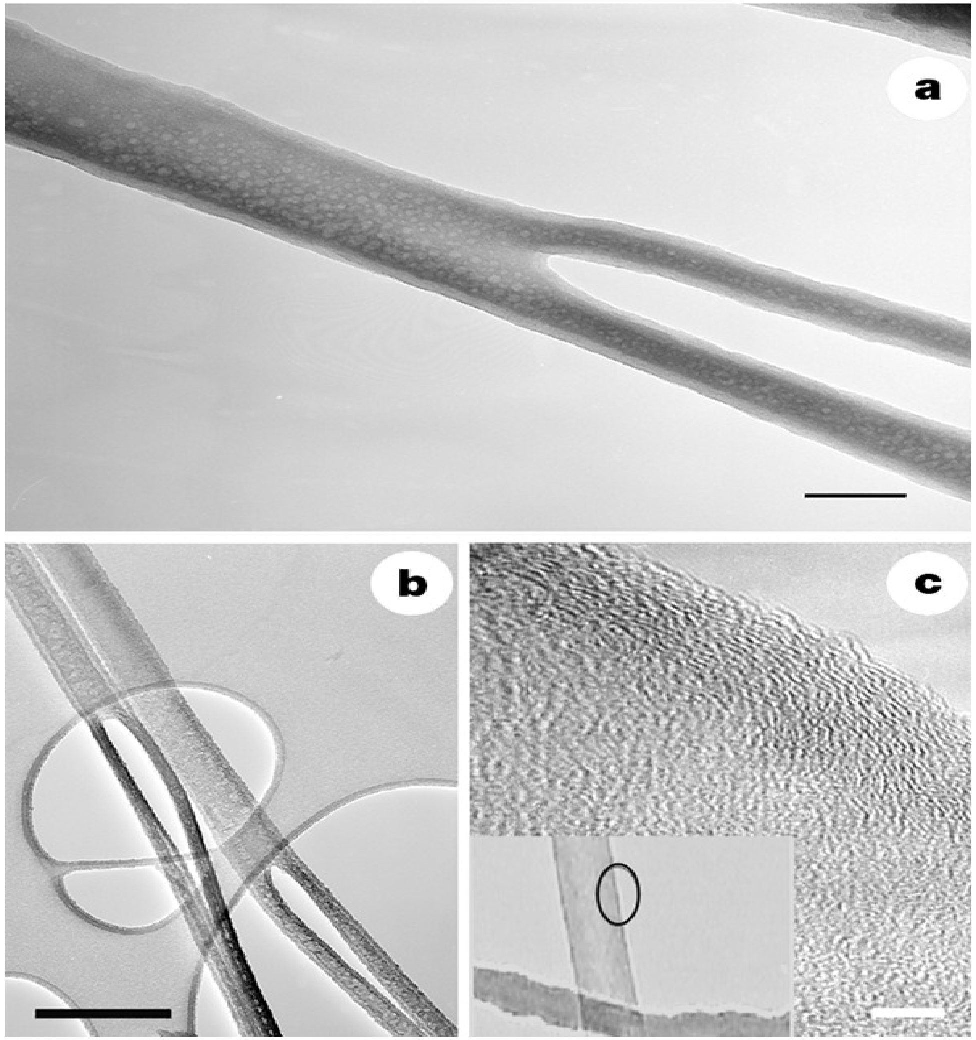
Fuhrer et al., Science 2000

Fusion: Electron beam welding
(transmission electron microscope)



Terrones et al., PRL 2002

Nanotube Y junctions

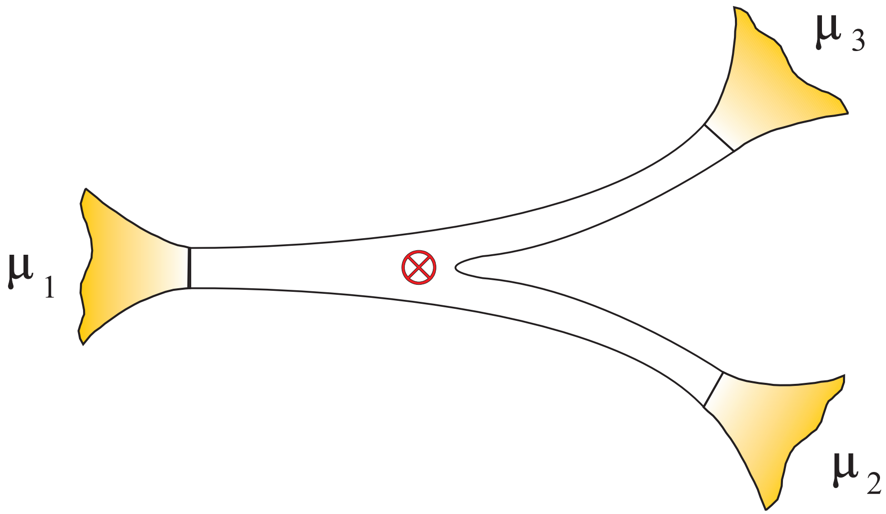


Li et al., Nature 1999

Landauer-Büttiker type theory for Luttinger liquids?

- Standard scattering approach useless:
  - Elementary excitations are fractionalized quasiparticles, not electrons
  - No simple scattering of electrons, neither at junction nor at contact to reservoirs
- Generalization to Luttinger liquids
  - Coupling to reservoirs via radiative boundary conditions (or $g(x)$ approach)
  - Junction: Boundary condition plus impurities

Description of junction (node)



Chen, Trauzettel & Egger, PRL 2002
Egger, Trauzettel, Chen & Siano, NJP 2003

- Landauer-Büttiker: Incoming and outgoing states related via scattering matrix $$\Psi_{out}(0) = S\Psi_{in}(0)$$
- Difficult to handle for correlated systems
- What to do ?

Some recent proposals …

- Perturbation theory in interactions

  Lal, Rao & Sen, PRB 2002
- Perturbation theory for almost no transmission *Safi, Devillard & Martin, PRL 2001*
- Node as island *Nayak, Fisher, Ludwig & Lin, PRB 1999*
- Node as ring *Chamon, Oshikawa & Affleck, PRL 2003*
- Node boundary condition for ideal symmetric junction (exactly solvable)
  - additional impurities generate arbitrary *S* matrices, no conceptual problem *Chen, Trauzettel & Egger, PRL 2002*

Ideal symmetric junctions

- *N>2* branches, junction with *S* matrix

$$S = \begin{pmatrix} z-1 & z & \dots & z \\ z & z-1 & \dots & z \\ \dots & \dots & \dots & \dots \\ z & z & \dots & z-1 \end{pmatrix} \qquad z = \frac{2}{N+i\lambda}, \lambda \geq 0$$

Crossover from full to no transmission tuned by λ

Texier & Montambaux, JP A 2001

- implies wavefunction matching at node

$$\Psi_1(0) = \Psi_2(0) = \dots = \Psi_N(0)$$

$$\Psi_j(0) = \Psi_{j,in}(0) + \Psi_{j,out}(0)$$

Boundary conditions at the node

- Wavefunction matching implies density matching $\rho_1(0) = \ldots = \rho_N(0)$

  → can be handled for Luttinger liquid

- Additional constraints:
  - Kirchhoff node rule $\sum_i I_i = 0$
  - Gauge invariance

- Nonlinear conductance matrix can then be computed exactly for arbitrary parameters

$$G_{ij} = \frac{e}{h} \frac{\partial I_i}{\partial \mu_j}$$

Solution for Y junction with *g*=1/2

Nonlinear conductance:

$$G_{ii} = \frac{8}{9}\left(1 - {\partial V_i}\big/{\partial U_i}\right) + \frac{2}{9}\sum_{j\neq i}\left(1 - {\partial V_j}\big/{\partial U_j}\right)$$

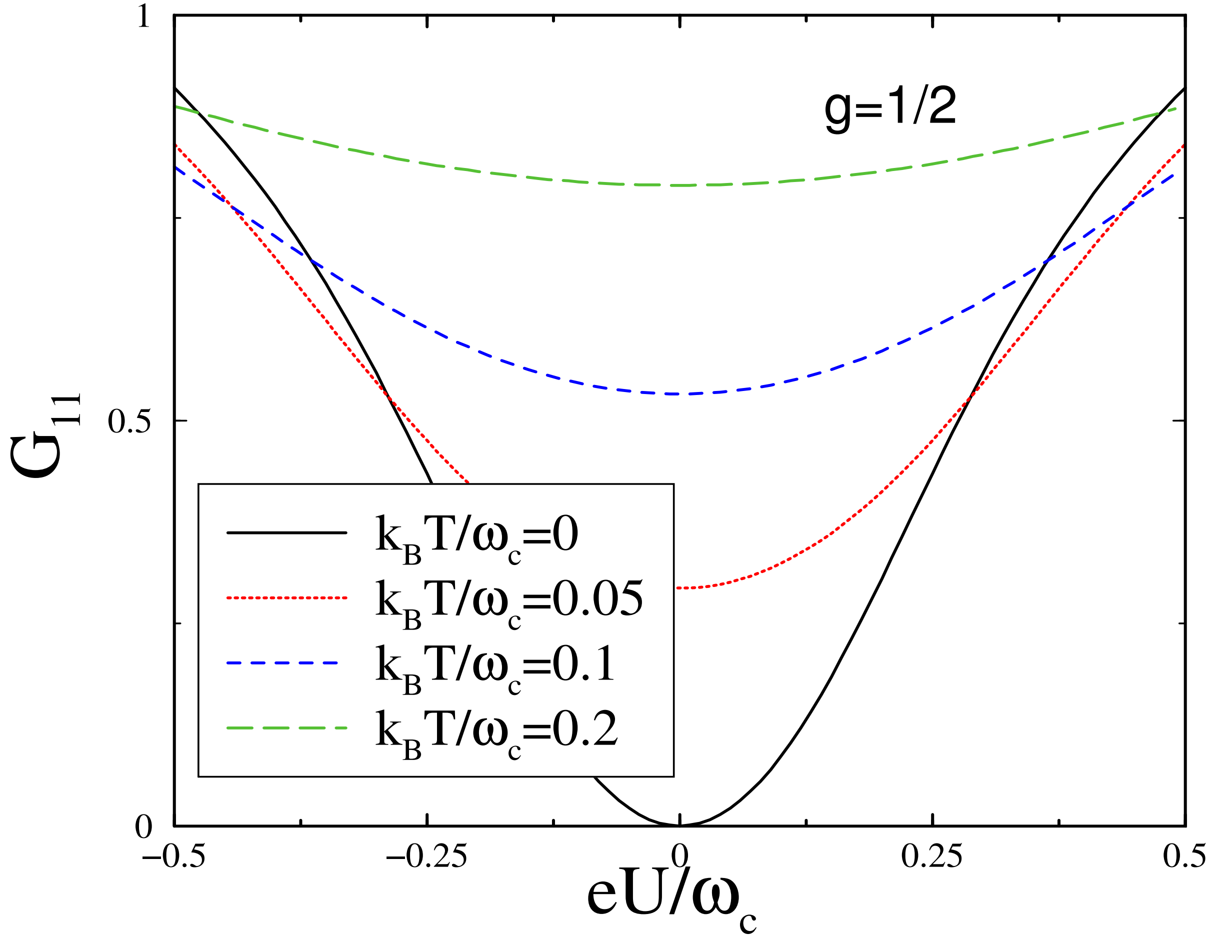
with

$$\frac{eV_i}{2T_B} = \operatorname{Im}\Psi\left(\frac{1}{2} + \frac{T_B + ie\left(U_i - V_i/2\right)}{2\pi T}\right)$$

$$T_B / D = w_0^{1/(1-g)}$$

$$w_0(N,\lambda) = \frac{2\left(\sqrt{N^2+\lambda^2} - \sqrt{2N}\right)}{\sqrt{N(N-2)+\lambda^2}}$$

Nonlinear conductance

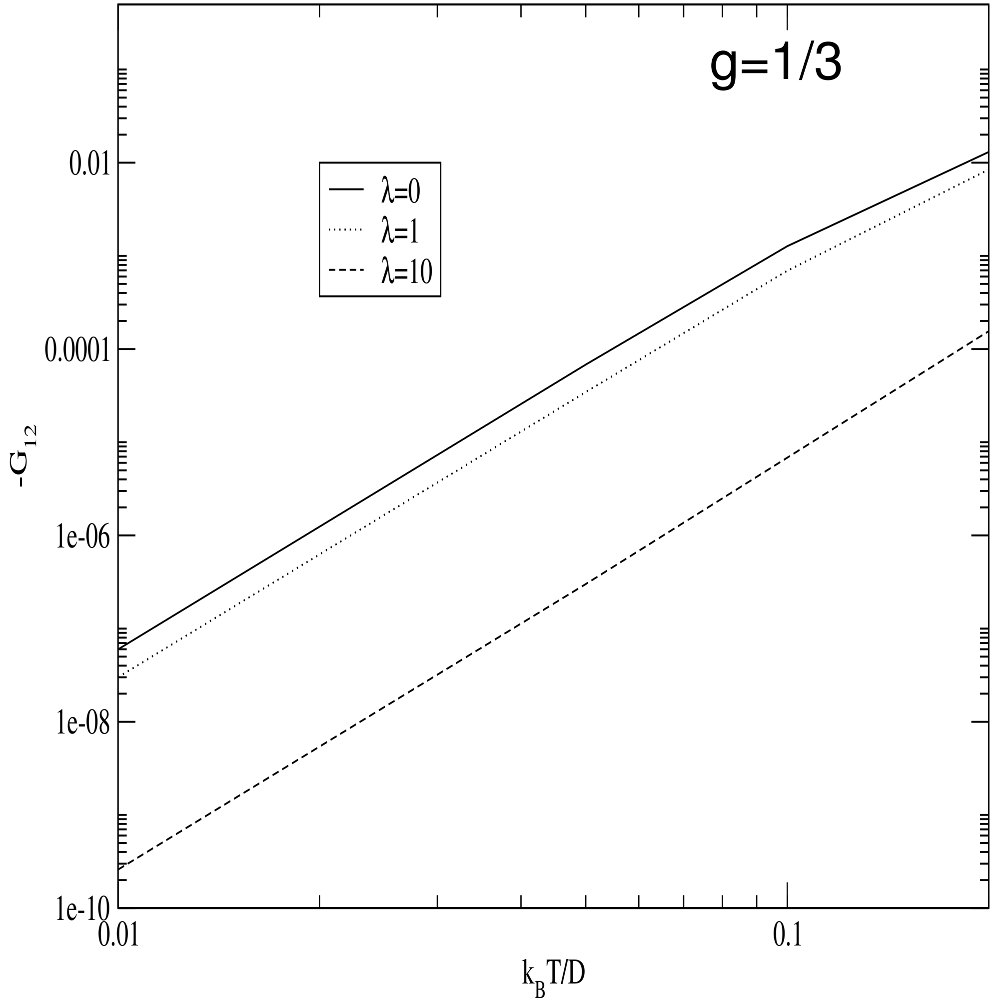


$$\mu_1 = \varepsilon_F + eU$$

$$\mu_2 = \mu_3 = \varepsilon_F$$

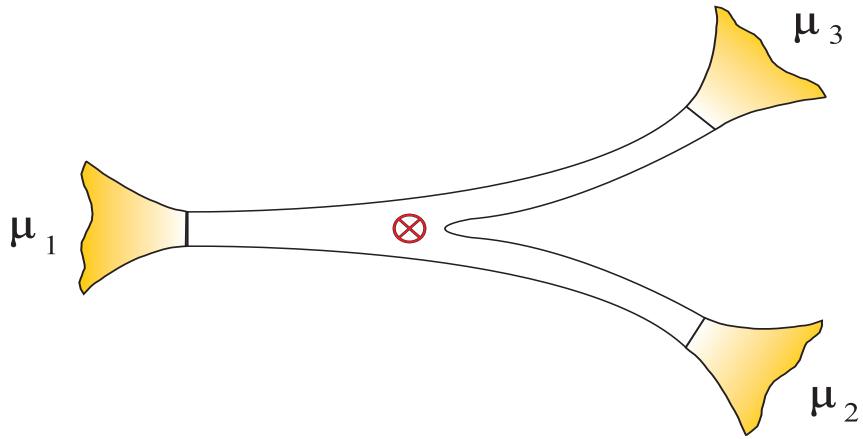
Ideal junction: Fixed point

- Symmetric system breaks up into disconnected wires at low energies
- Only stable fixed point
- Typical Luttinger power law for all conductance coefficients



Asymmetric Y junction

- Add one impurity of strength *W* in tube 1 close to node
- Exact solution possible for *g=3/8* (Toulouse limit in suitable rotated picture)
- Transition from truly insulating node to disconnected tube 1 + perfect wire 2+3



Asymmetric Y junction: *g=3/8*

- Full solution:

$$I_1 = I_1^0 - \delta I , I_{2,3} = I_{2,3}^0 + \delta I / 2$$

- Asymmetry contribution

$$\pi \delta I = eW_B \operatorname{Im} \Psi \left(\frac{1}{2} + \frac{W_B + 2\pi i \left[I_1^0 - \delta I / 2 \right] / e}{2\pi T} \right)$$

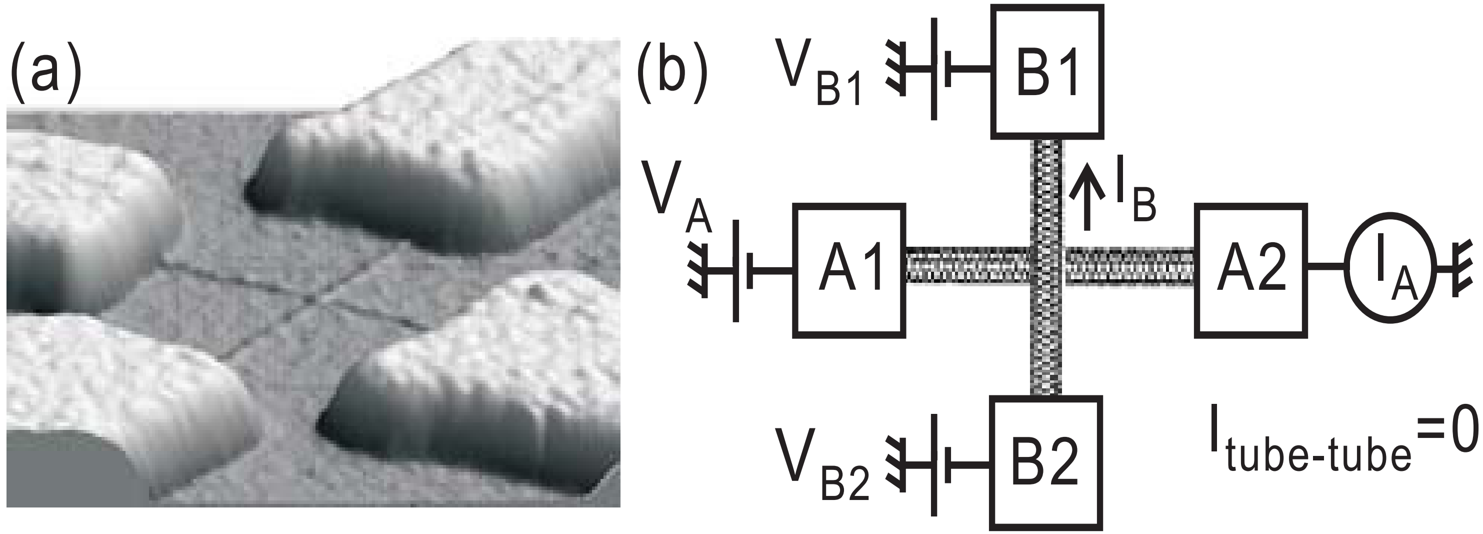
$$W_B = \pi W^2 / D$$

- Strong asymmetry limit:

$$I_1 = 0, I_{2,3} = I_{2,3}^0 + I_1^0 / 2$$

Crossed tubes: Theory vs. experiment

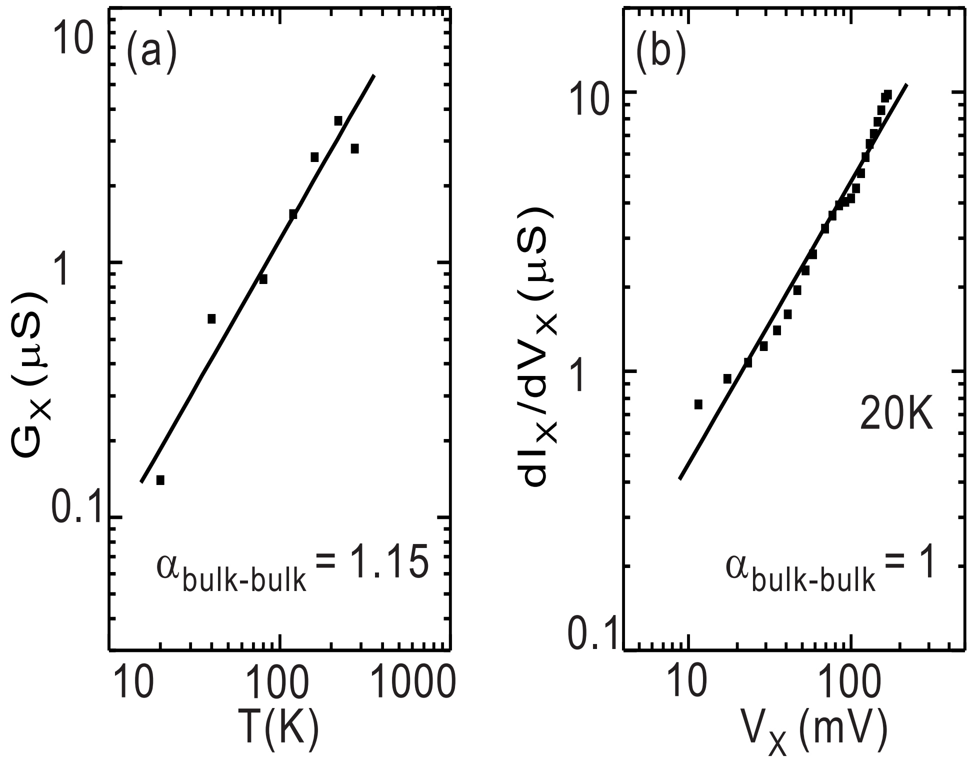
Komnik & Egger, PRL 1998, EPJB 2001
Gao, Komnik, Egger, Glattli, Bachtold, PRL 2004



- Weakly coupled crossed nanotubes
  - Single-electron tunneling between tubes irrelevant
  - Electrostatic coupling relevant for strong interactions
- Without tunneling: Local Coulomb drag

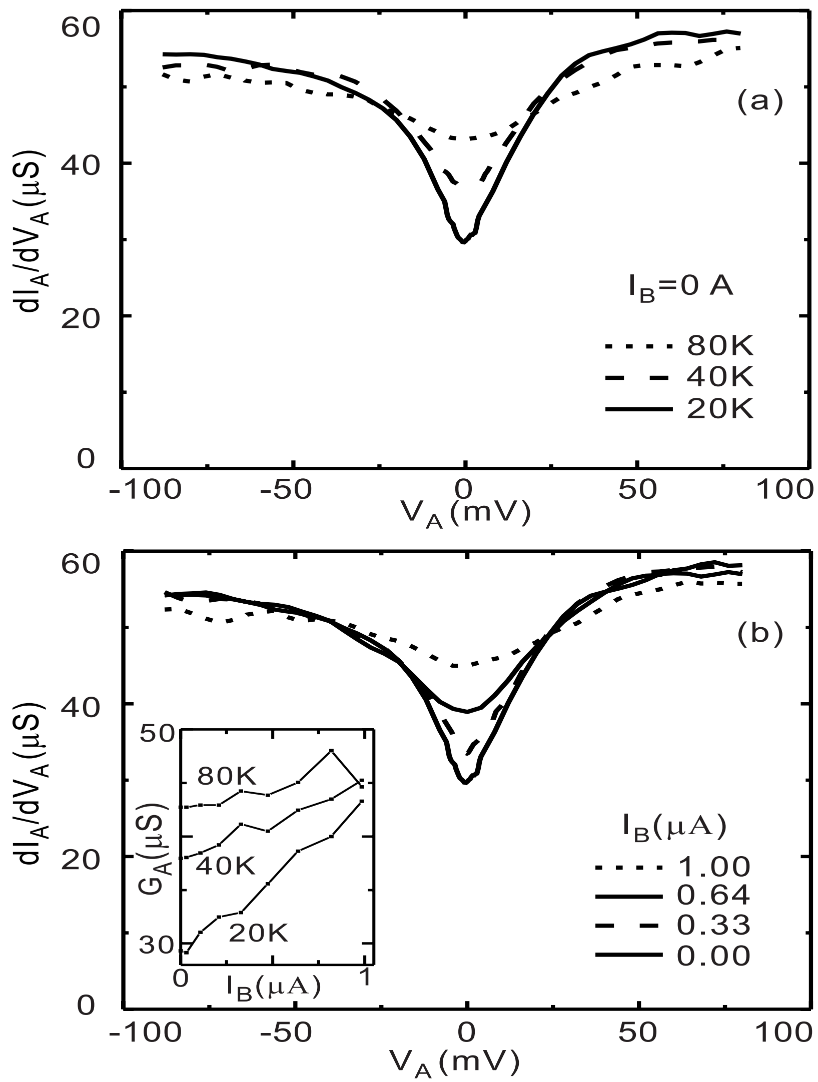
Characterization: Tunneling DoS

- Tunneling conductance through crossing: Power law, consistent with Luttinger liquid
- Quantitative fit gives *g=0.16*
- Evidence for Luttinger liquid beyond TDoS?



Dependence on transverse current

- Experimental data show suppression of zero-bias anomaly when current flows through transverse tube
- Coulomb blockade or heating mechanisms can be ruled out
- Prediction of Luttinger liquid theory?



Hamiltonian for crossed tubes

- Without tunneling: Electrostatic coupling and crossing-induced backscattering

$$H = H_0^A + H_0^B + \lambda_0 \rho_A(0) \rho_B(0) + \sum_{i=A/B} \lambda_i \rho_i(0)$$

$$H_0^i = \frac{1}{2} \int dx \left[\Pi_i^2 + (\partial_x \varphi_i)^2 \right]$$

- Density operator:

$$\rho_{A/B}(x) \propto \cos \left[\sqrt{16 \pi g}\, \varphi_{A/B}(x) \right]$$

Renormalization group equations

- Lowest-order RG equations:

$$\frac{d\lambda_0}{dl} = (1 - 8g)\lambda_0 + 2\lambda_A\lambda_B$$

$$\frac{d\lambda_{A/B}}{dl} = (1 - 4g)\lambda_{A/B}$$

- Solution:

$$\lambda_{A/B}(l) = e^{(1-4g)l}\lambda_{A/B}(0)$$

$$\lambda_0(l) = e^{(1-8g)l}\left[\lambda_0(0) - 2\lambda_A(0)\lambda_B(0)\right] + 2e^{(2-8g)l}\lambda_A(0)\lambda_B(0)$$

- Here: inter-tube coupling most relevant!

Low-energy solution

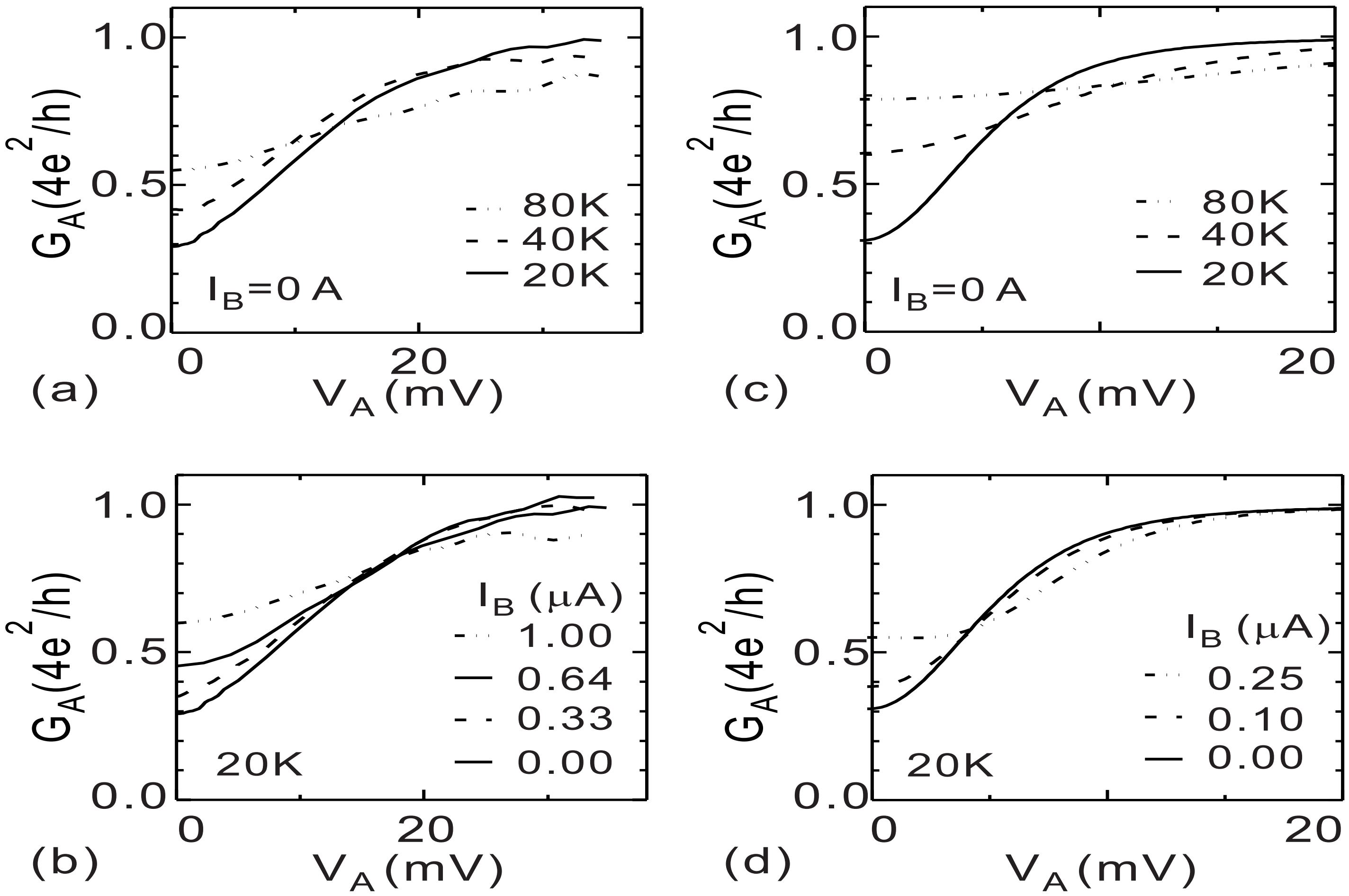
- Keeping only inter-tube coupling, problem is exactly solvable by switching to symmetric and antisymmetric (±) boson fields
- For *g=3/16=0.1875*, particularly simple:

$$I_{A/B} = \frac{4e^2}{h}\left[V_{A/B} - \frac{U_+ \pm U_-}{\sqrt{2}}\right]$$

$$eU_\pm = 2k_B T_B \,\mathrm{Im}\,\Psi\left(\frac{1}{2} + \frac{k_B T_B + ie\left(V_\pm - U_\pm\right)}{2\pi k_B T}\right)$$

$$V_\pm = \frac{V_A \pm V_B}{\sqrt{2}}$$

Comparison to experimental data



Experimental data

Theory

New evidence for Luttinger liquid

Gao, Komnik, Egger, Glattli & Bachtold, PRL 2004

- Rather good agreement, only one fit parameter: $T_B = 11.6\,K$
- No alternative explanation works
- Agreement is taken as new evidence for Luttinger liquid in nanotubes, beyond previous tunneling experiments
- Additional evidence from photoemission experiments *Ishii et al., Nature 2003*

Coulomb drag: Transconductance

- Strictly local coupling: Linear transconductance G_{21} always vanishes
- Finite length: Couplings in +/- sectors differ

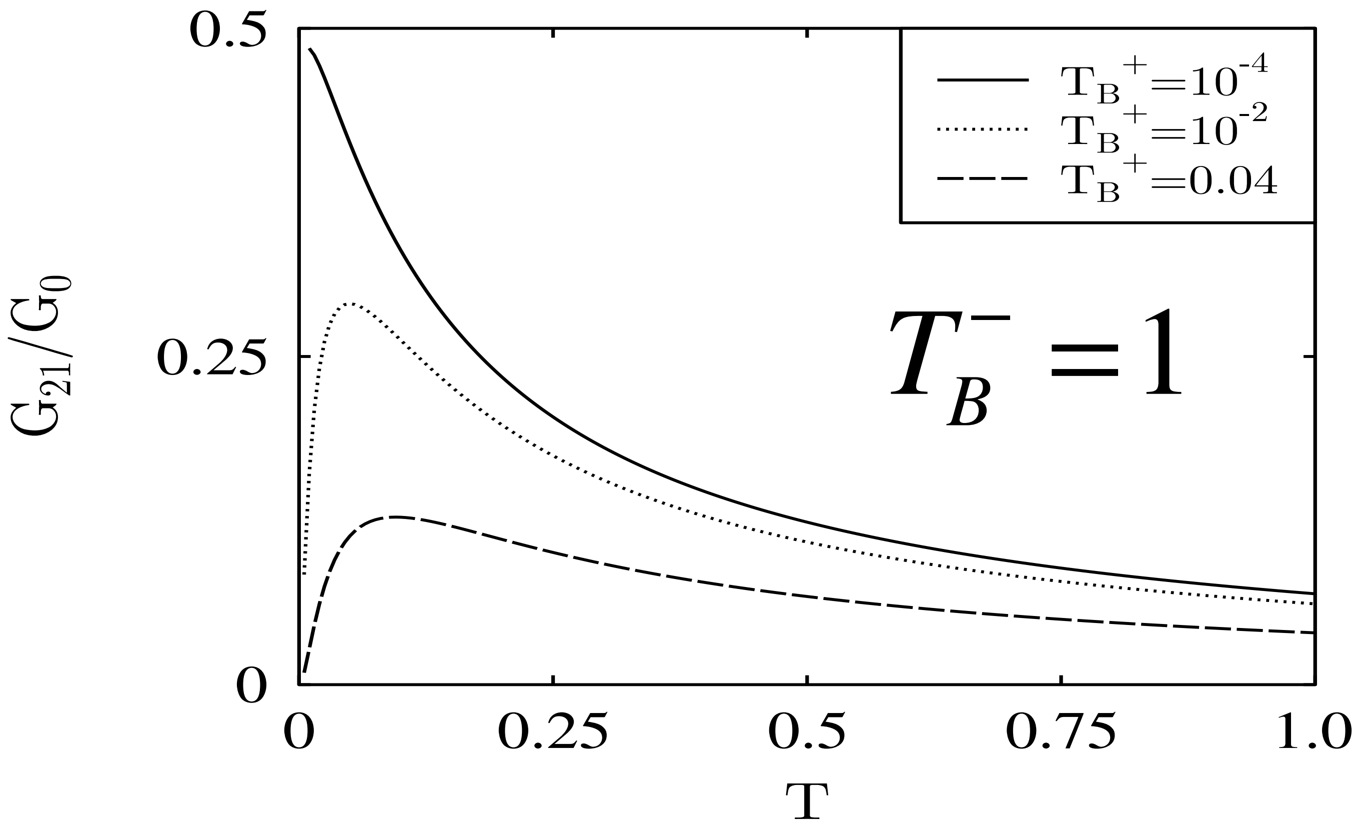
$$\lambda_0 \to \lambda_\pm = \frac{\lambda_0}{L} \int_{-L/2}^{L/2} dx \cos\left[2(k_{F,A} \pm k_{F,B})x\right]$$

$$T_B^\pm / D = \left(\lambda_\pm \big/ D \right)^{1/(1-2g)}$$

$$T_B^+ \leq T_B^-$$

Now nonzero linear transconductance, except at *T=0*!

Linear transconductance: *g=1/4*



$$G_{21} = \frac{1}{2}\sum_{\pm} \pm \frac{1 - c_{\pm}\Psi'_{\pm}(c_{\pm} + 1/2)}{1 + c_{\pm}\Psi'_{\pm}(c_{\pm} + 1/2)}$$

$$c_{\pm} = T_B^{\pm} / 2\pi T$$

Absolute Coulomb drag

Averin & Nazarov, PRL 1998
Flensberg, PRL 1998
Komnik & Egger, PRL 1998, EPJB 2001

For long contact & low temperature (but finite): Transconductance approaches maximal value

$$G_{21}(T \neq 0, T_B^+ / T_B^- \to 0) = \frac{e^2 / h}{2}$$

Coulomb drag shot noise

Trauzettel, Egger & Grabert, PRL 2002

- Shot noise at T=0 gives important information beyond conductance

$$P(\omega) = \int dt e^{i\omega t} \langle \delta I(t) \delta I(0) \rangle$$

- For two-terminal setup & one weak impurity: DC shot noise carries no information about fractional charge

$$P = 2eI_{BS}$$

Ponomarenko & Nagaosa, PRB 1999

- Crossed nanotubes: For $V_A = 0, V_B \neq 0 \Rightarrow P_A \neq 0$ must be due to cross voltage (drag noise)

Shot noise transmitted to other tube

- Mapping to decoupled two-terminal problems in ± channels implies $\left\langle \delta I_+(t)\,\delta I_-(0)\right\rangle = 0$

- Consequence: Perfect shot noise locking

$$P_A = P_B = (P_+ + P_-)/2$$

  - Noise in tube A due to cross voltage is exactly equal to noise in tube B
  - Requires strong interactions, $g<1/2$
  - Effect survives thermal fluctuations

Multi-wall nanotubes: The disorder-interaction problem

- Russian doll structure, electronic transport in MWNTs usually in outermost shell only
- Energy scales one order smaller
- Typically $N_{bands} \approx 20$ due to doping
- Inner shells can also create `disorder´
  - Experiments indicate mean free path $\ell \approx R \ldots 10R$
  - Ballistic behavior on energy scales

$$E\tau > 1, \tau = \ell / v_F$$

Tunneling between shells

Maarouf, Kane & Mele, PRB 2001

- Bulk 3D graphite is a metal: Band overlap, tunneling between sheets quantum coherent
- In MWNTs this effect is strongly suppressed
  - Statistically 1/3 of all shells metallic (random chirality), since inner shells undoped
  - For adjacent metallic tubes: Momentum mismatch, incommensurate structures
  - Coulomb interactions suppress single-electron tunneling between shells

Interactions in MWNTs: Ballistic limit

Egger, PRL 1999

- Long-range tail of interaction unscreened
- Luttinger liquid survives in ballistic limit, but Luttinger exponents are close to Fermi liquid, e.g.

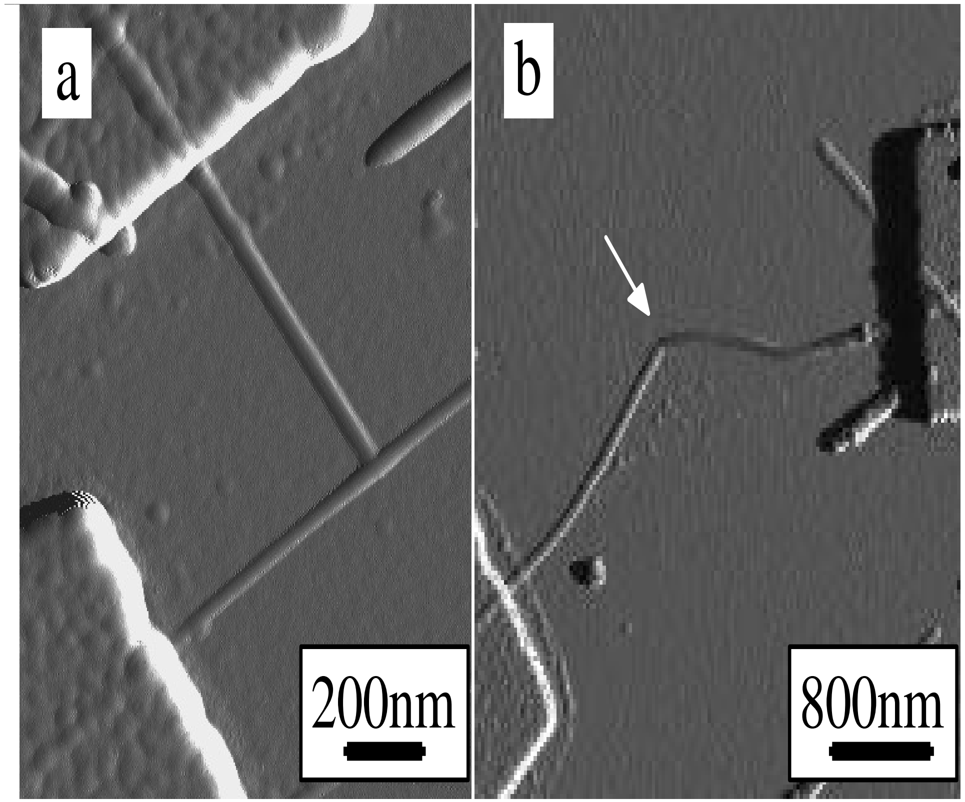
$$\eta \propto 1 / \sqrt{N_{bands}}$$

- End/bulk tunneling exponents are at least one order smaller than in SWNTs
- Weak backscattering corrections to conductance suppressed even more!

Experiment: TDoS of MWNT

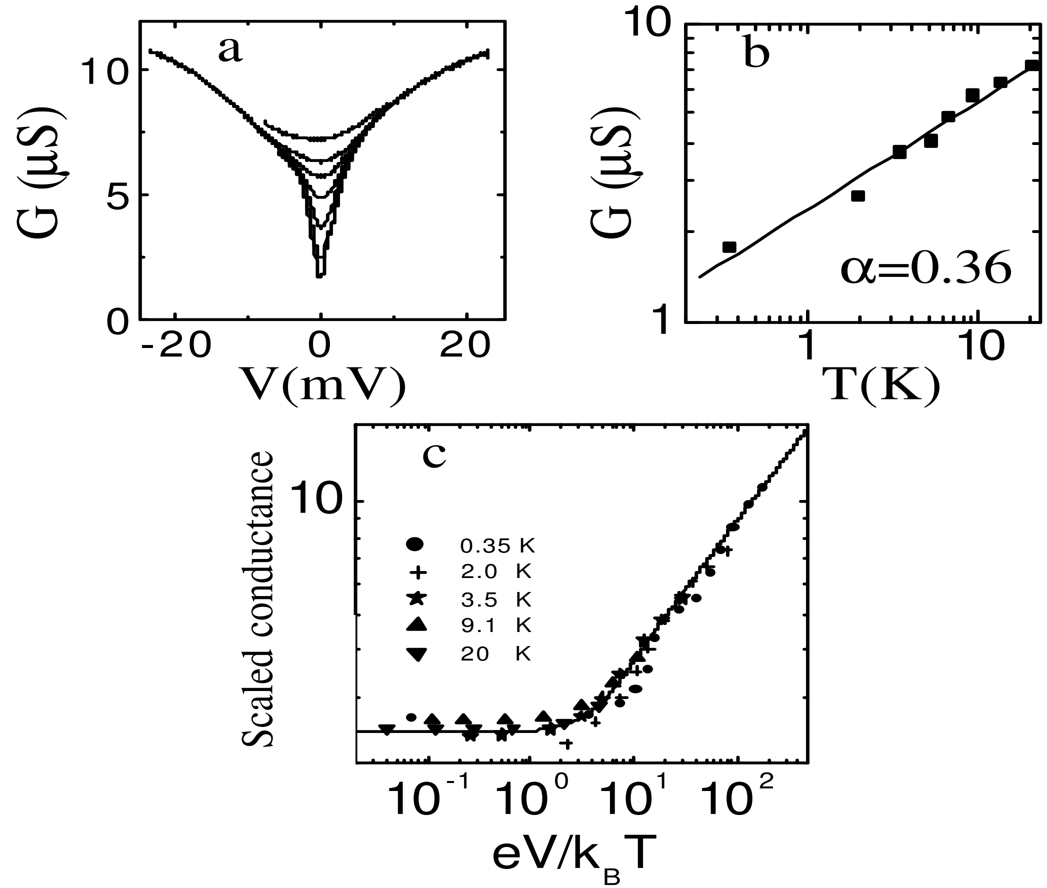
Bachtold et al., PRL 2001

- TDoS observed from conductance through tunnel contact
- Power law zero-bias anomalies
- Scaling properties similar to a Luttinger liquid, but: exponent larger than expected from Luttinger theory

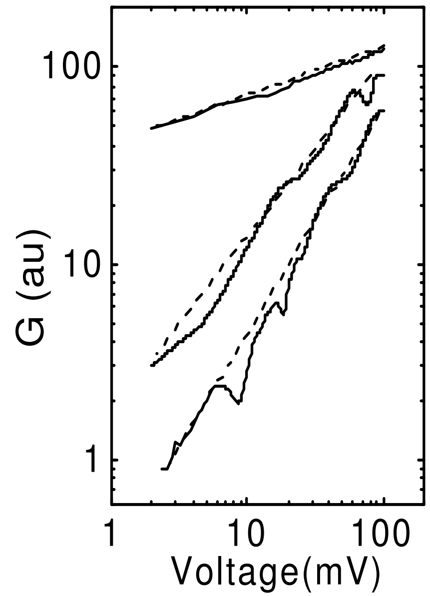


Tunneling DoS of MWNTs

Bachtold et al., PRL 2001



Geometry dependence



$$\eta_{end} = 2\eta_{bulk}$$

Interplay of disorder and interaction

Egger & Gogolin, PRL 2001
Mishchenko, Andreev & Glazman, PRL 2001

- Coulomb interaction enhanced by disorder
- Nonperturbative theory: Interacting Nonlinear σ Model *Kamenev & Andreev, PRB 1999*
- Equivalent to Coulomb Blockade: spectral density *I(ω)* of intrinsic electromagnetic modes

$$P(E) = \mathrm{Re}\int_0^\infty \frac{dt}{\pi}\exp\left[iEt + J(t)\right]$$

$$J(T=0,t) = \int_0^\infty \frac{d\omega}{\omega} I(\omega)\left(e^{-i\omega t} - 1\right)$$

Intrinsic Coulomb blockade

- TDoS ⟺ Debye-Waller factor *P(E)*:

$$\frac{\nu(E)}{\nu_0} = \int d\varepsilon P(E-\varepsilon)\frac{1+e^{-E/k_BT}}{1+e^{-\varepsilon/k_BT}}$$

- For constant spectral density: Power law with exponent $\alpha = I(\omega \to 0)$ Here:

$$I(\omega) = \frac{U_0}{2\pi(D^* - D)} \mathrm{Re}\sum_n \left(\left[-i\omega / D^* + n^2/R^2 \right]^{-1/2} - \left(D^* \to D\right) \right)$$

$$D^*/D = 1 + \nu_0 U_0,\ D = v_F^2 \tau / 2$$

Field/particle diffusion constants

Dirty MWNT

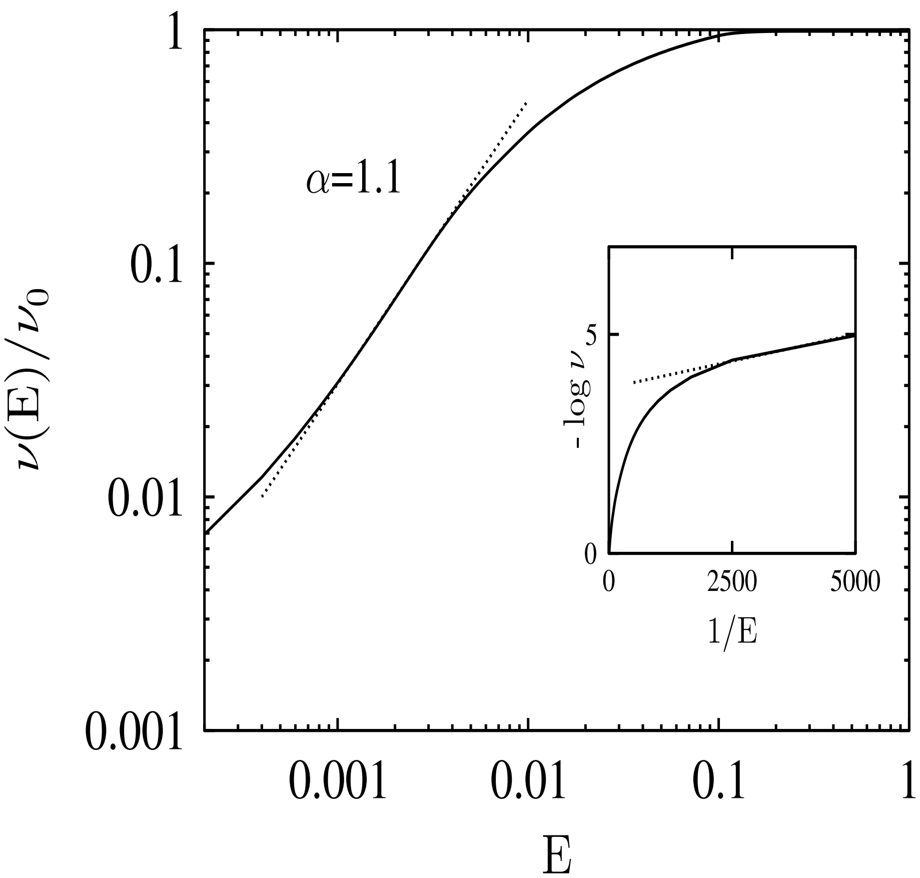
- High energies: $E > E_{Thouless} = D/(2\pi R)^2$
- Summation can be converted to integral, yields constant spectral density, hence power law TDoS with $\alpha = \frac{R}{2\pi\nu_0 D}\ln\left(D^*/D\right)$
- Tunneling into interacting diffusive 2D metal
- Altshuler-Aronov law exponentiates into power law. But: restricted to $\ell < R$

Numerical solution

Egger & Gogolin, Chem.Phys.2002

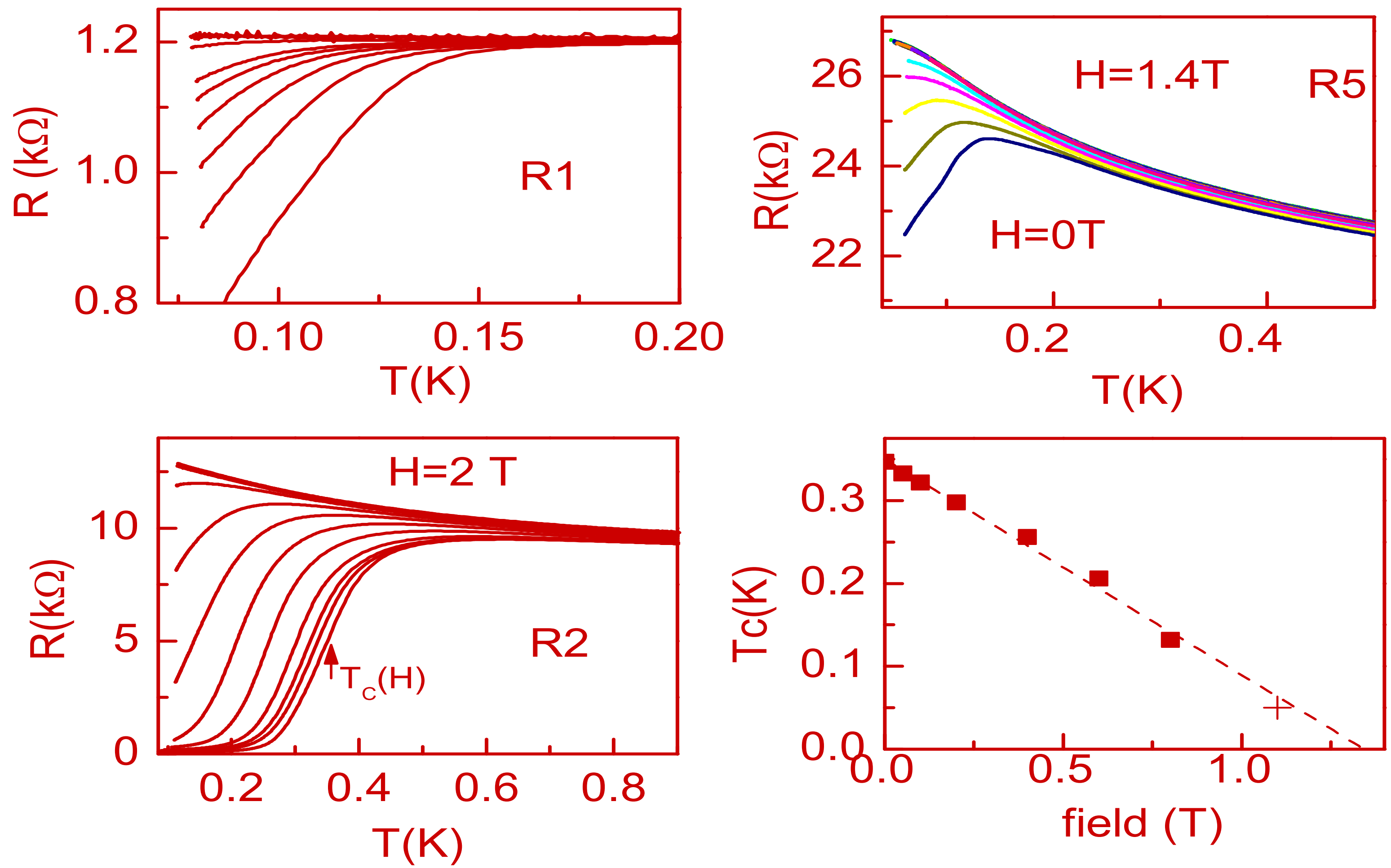
- Power law well below Thouless scale
- Smaller exponent for weaker interactions, only weak dependence on mean free path
- 1D pseudogap at very low energies

Mishchenko et al., PRL 2001



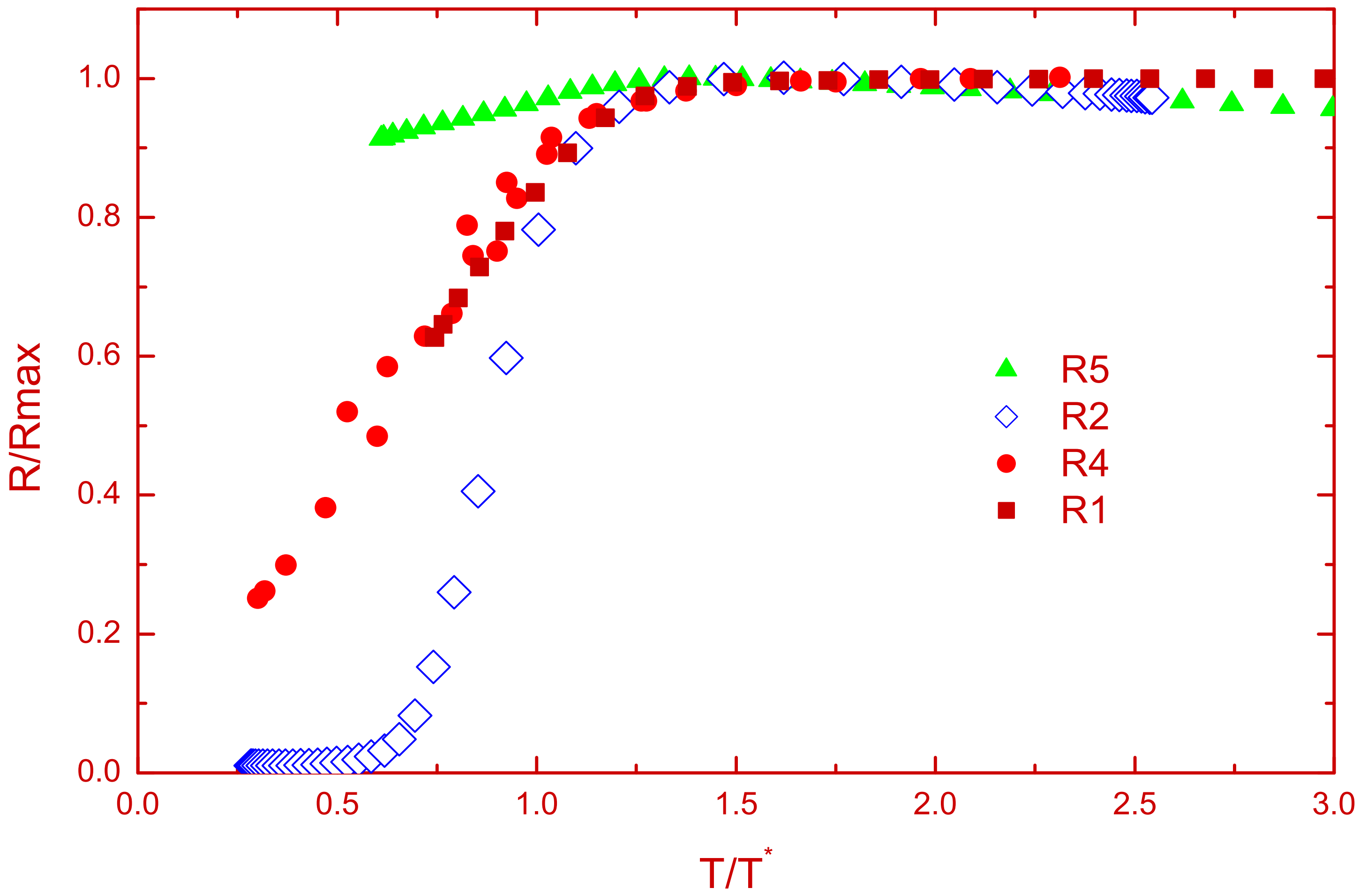
$$\ell = 10R, U_0 / 2\pi v_F = 1, v_F / R = 1$$

Superconductivity in ropes of SWNTs



Kasumov et al., PRB 2003

Experimental results for resistance



Kasumov et al., PRB 2003

Continuum elastic theory of a SWNT: Acoustic phonons

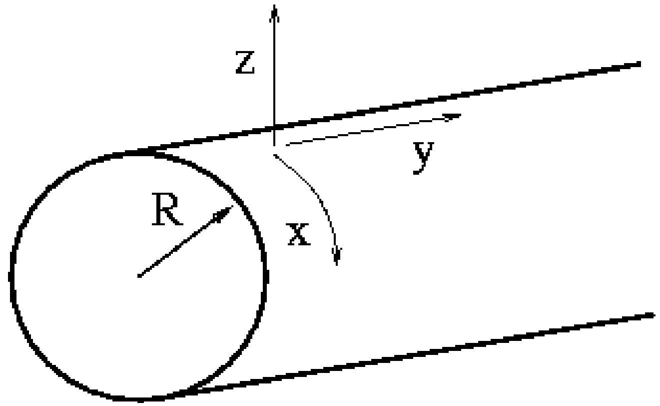
De Martino & Egger, PRB 2003

- Displacement field: $\vec{u}(x,y) = (u_x, u_y, u_z)$
- Strain tensor:

$$u_{yy} = \partial_y u_y$$

$$u_{xx} = \partial_x u_x + u_z / R$$

$$2u_{xy} = \partial_y u_x + \partial_x u_y$$



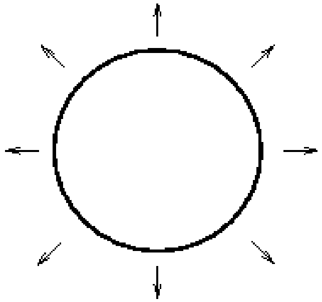
- Elastic energy density:

$$U(\vec{u}) = \frac{B}{2}\left(u_{xx} - u_{yy}\right)^2 + \frac{\mu}{2}\left(\left(u_{xx} - u_{yy}\right)^2 + 4u_{xy}^2\right)$$

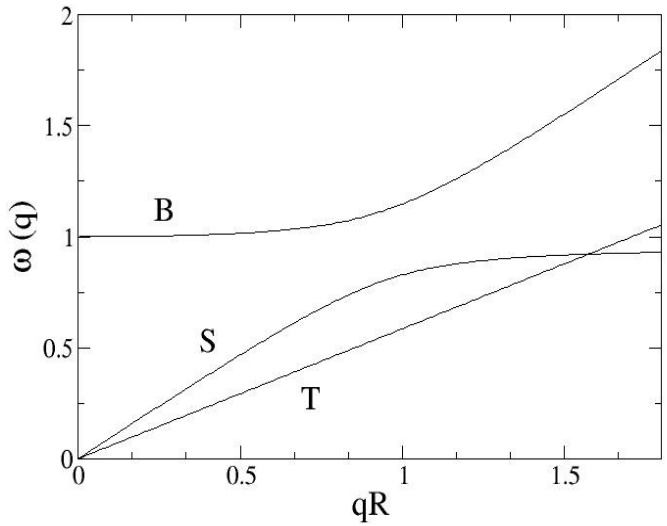
Suzuura & Ando, PRB 2002

Normal mode analysis

- Breathing mode

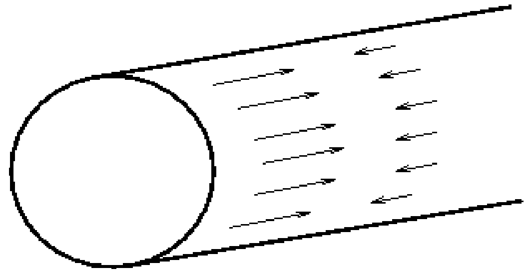


$$\omega_B = \sqrt{\frac{B+\mu}{MR^2}} \approx \frac{0.14}{R} \text{eV Å}$$



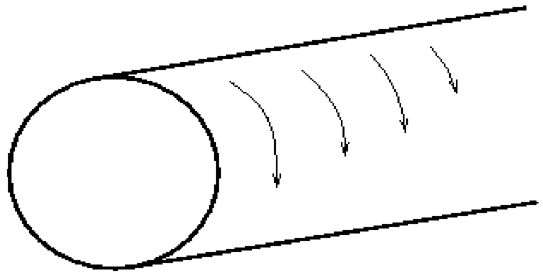
- Stretch mode

$$v_S = \sqrt{4B\mu / M(B+\mu)} \approx 2\times10^4 \text{ m/s}$$



- Twist mode

$$v_T = \sqrt{\mu / M} \approx 1.2\times10^4 \text{ m/s}$$



Electron-phonon coupling

- Main contribution from deformation potential

$$V(x,y) = \alpha\left(u_{xx} + u_{yy}\right) \qquad \alpha \approx 20-30\,\mathrm{eV}$$

couples to electron density

$$H_{el-ph} = \int dxdy\ V\rho$$

- Other electron-phonon couplings small, but potentially responsible for Peierls distortion
- Effective electron-electron interaction generated via phonon exchange (integrate out phonons)

SWNTs with phonon-induced interactions

- Luttinger parameter in one SWNT due to screened Coulomb interaction: $g = g_0 \leq 1$
- Assume good screening (e.g. thick rope)
- Breathing-mode phonon exchange causes attractive interaction:

$$g = \frac{g_0}{\sqrt{1 - g_0^2 R_B / R}}$$

For (10,10) SWNT:

$$g \approx 1.3 > 1$$

$$R_B = \frac{2\alpha^2}{\pi^2 v_F (B + \mu)} \approx 0.24 nm$$

Superconductivity in ropes

De Martino & Egger, PRB 2004

Model:

$$H = \sum_{i=1}^{N} H_{Lutt}^{(i)} - \sum_{ij} \Lambda_{ij} \int dy\, \Theta_i^* \Theta_j$$

- Attractive electron-electron interaction within each of the *N* metallic SWNTs
- Arbitrary Josephson coupling matrix, keep only singlet on-tube Cooper pair field $\Theta_i(y,\tau)$
- Single-particle hopping again negligible

Order parameter for nanotube rope superconductivity

- Hubbard Stratonovich transformation: complex order parameter field

$$\Delta_i(y,\tau) = |\Delta_i| e^{i\Phi_i}$$

to decouple Josephson terms

- Integration over Luttinger fields gives action:

$$S = \sum_{ij,y\tau} \Delta_i^* \Lambda_{ij}^{-1} \Delta_j - \ln \left\langle e^{-Tr\left(\Delta^* \Theta + \Theta^* \Delta\right)} \right\rangle_{Lutt}$$

Quantum Ginzburg Landau (QGL) theory

- 1D fluctuations suppress superconductivity
- Systematic cumulant & gradient expansion:
  Expansion parameter $|\Delta| / 2\pi T$
- QGL action, coefficients from full model

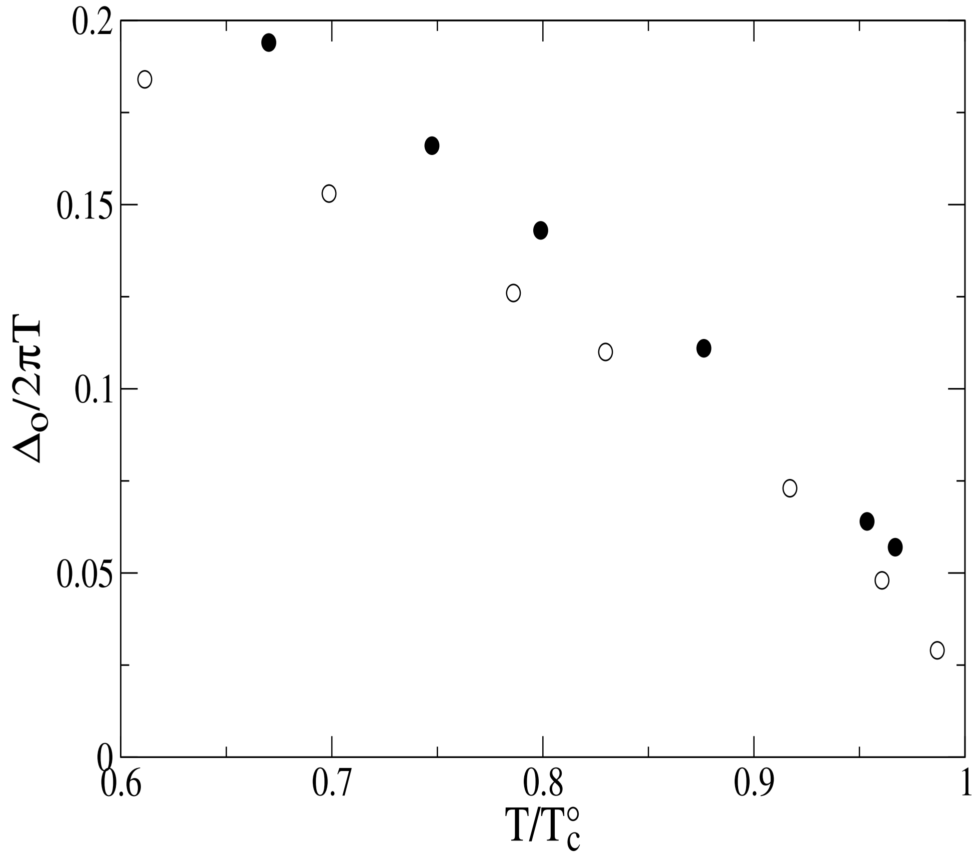
$$S = Tr\left\{\left(\Lambda_1^{-1} - A\right)|\Delta|^2 + B|\Delta|^4\right\} +$$
$$+ Tr\left\{C|\partial_y \Delta|^2 + D|\partial_\tau \Delta|^2\right\} +$$
$$+ Tr\sum_{ij} \Delta_i^* \left(\Lambda_{ij}^{-1} - \Lambda_1^{-1}\right)\Delta_j$$

Amplitude of the order parameter

- Mean-field transition at

$$A\left(T_c^{\,0}\right) = \Lambda_1$$

- For lower *T*, amplitudes are finite, with gapped fluctuations
- Transverse fluctuations irrelevant for $N \leq 100$
- QGL accurate down to very low T



Low-energy theory: Phase action

- Fix amplitude at mean-field value: Low-energy physics related to phase fluctuations

$$S = \frac{\mu}{2\pi}\int dy\,d\tau\left[c_s^{-1}(\partial_\tau\Phi)^2 + c_s(\partial_y\Phi)^2\right]$$

- Rigidity

$$\mu(T) = N\nu\left[1-\left(T/T_c^0\right)^{(g-1)/2g}\right]$$

$\nu \approx 1$ from QGL, but also influenced by dissipation or disorder

Quantum phase slips: Kosterlitz-Thouless transition to normal state

- Superconductivity can be destroyed by vortex excitations: Quantum phase slips (QPS)
- Local destruction of superconducting order allows phase to slip by *2π*
- QPS proliferate for $\mu(T) \leq 2$
- True transition temperature

$$T_c = T_c^0 \left[1 - \frac{2}{N\nu} \right]^{2g/(g-1)} \approx 0.1...0.5K$$

Resistance in superconducting state

De Martino & Egger, PRB 2004

- QPS-induced resistance
- Perturbative calculation, valid well below transition:

$$\frac{R(T)}{R(T_c)} = \left(T / T_c \right)^{2\mu(T)-3} \frac{\int_0^\infty du \frac{1}{1+u^2} \left| \frac{\Gamma(\mu/2 + iuT_L/2T)}{\Gamma(\mu/2)} \right|^4}{\int du \frac{1}{1+u^2} \left| \frac{\Gamma(\mu/2 + iuT_L/2T_c)}{\Gamma(\mu/2)} \right|^4}$$

$$T_L = c_s / \pi L$$

Comparison to experiment

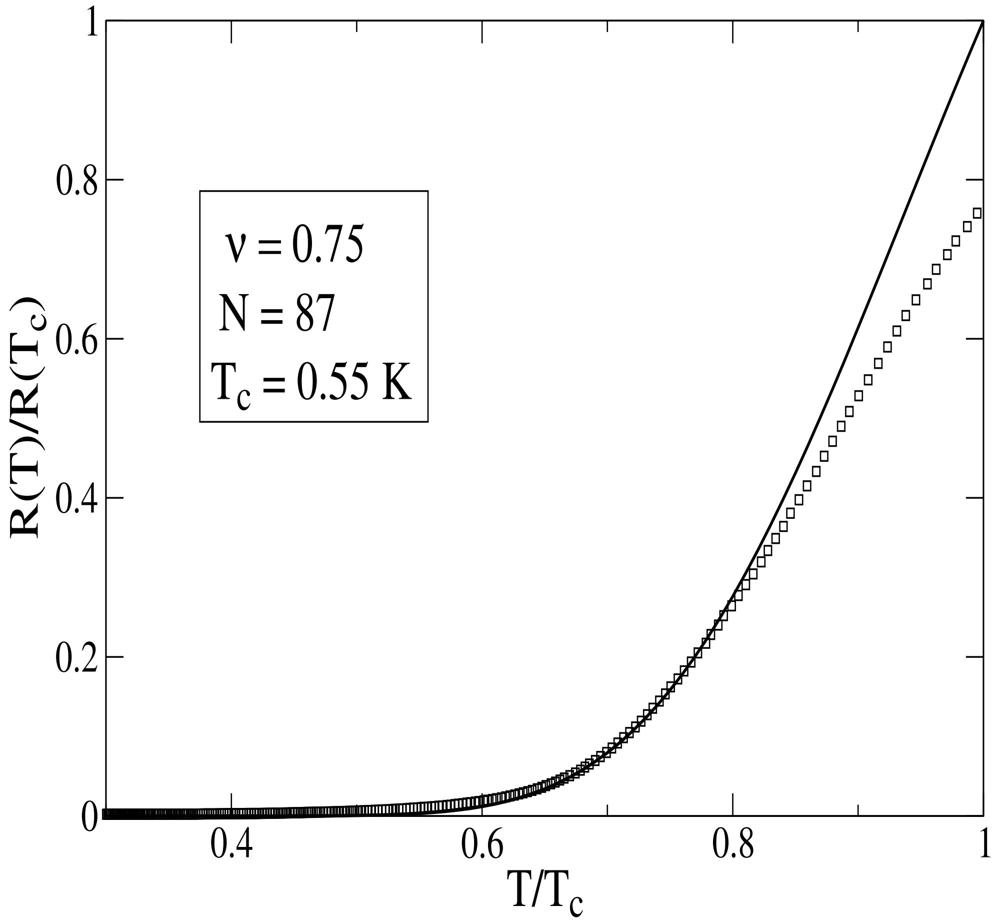
Ferrier, De Martino et al., Sol. State Comm. 2004

- Resistance below transition allows detailed comparison to Orsay experiments
- Free parameters of the theory:
  - Interaction parameter, taken as $g = 1.3$
  - Number *N* of metallic SWNTs, known from residual resistance (contact resistance)
  - Josephson matrix (only largest eigenvalue needed), known from transition temperature
  - Only one fit parameter remains: $\nu \approx 1$

Comparison to experiment: Sample R2

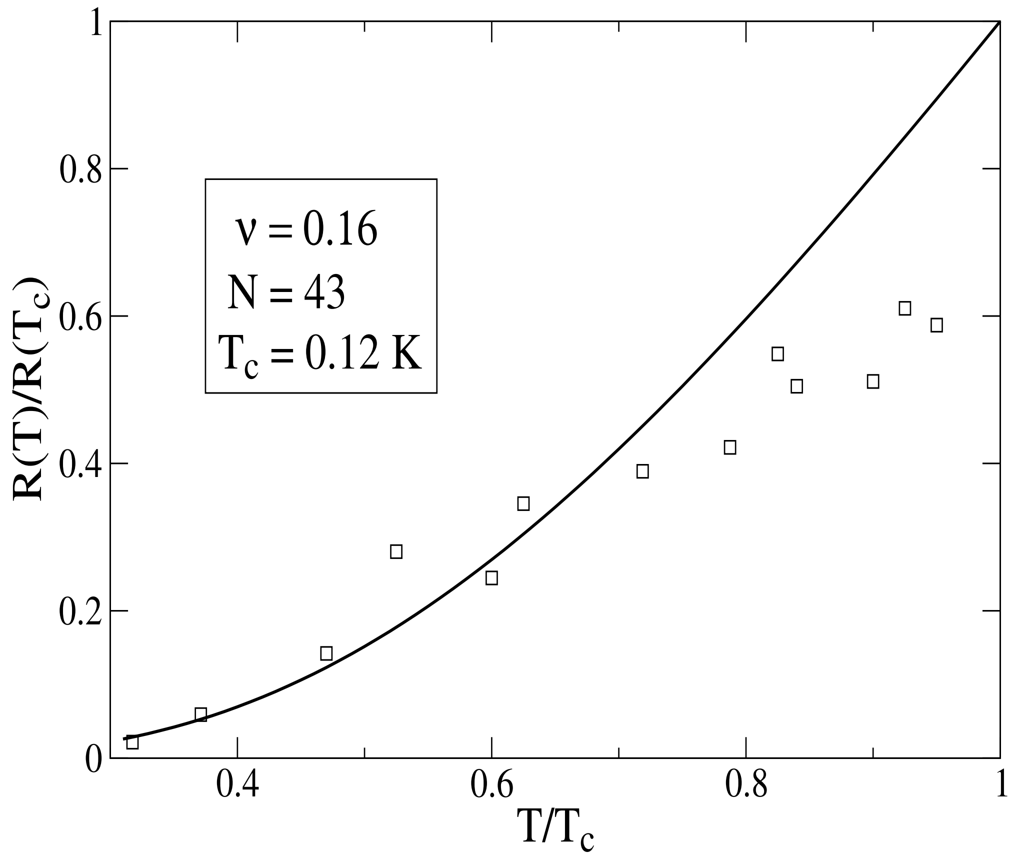
Nice agreement

- Fit parameter near 1
- Rounding near transition is not described by theory
- Quantum phase slips → low-temperature resistance
- Thinnest known superconductors



Comparison to experiment: Sample R4

- Again good agreement but more noise in experimental data
- Fit parameter now smaller than 1, dissipative effects
- Ropes of carbon nanotubes thus allow to observe quantum phase slips



Conclusions

- Nanotubes allow for field-theory approach
  - Bosonization & conformal field theory methods
  - Disordered field theories
- Close connection to experiments
  - Tunneling density of states
  - Crossed nanotubes & local Coulomb drag
  - Multiwall nanotubes
  - Superconductivity