Many-body phenomena in quasi-1D cold atom systems

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SFB Transregio 12

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Overview

- Two-body problem in 1D trap
  - Interacting cold gases in quasi-1D traps:
    experimentally accessible nontrivial many-body physics
  - Confinement-induced resonance and dimer state
  - 1D analogue of BCS-BEC crossover for two-species fermion gas
- Few- to many-body problem in confined geometry
  - Atom-dimer and dimer-dimer scattering
  - Many-body solution covering full crossover
- Disordered strongly interacting 1D Bose gas

Confinement: Two-body problem

Olshanii, PRL 1998

- Pseudopotential: s-wave scattering length

$$V(\vec{r}) = \frac{4\pi\hbar^2 a}{m_0}\delta(\vec{r})\frac{\partial}{\partial r}r$$

- Harmonic confinement

$$U_c(\vec{r}) = m_0\omega_\perp^2\left(x^2 + y^2\right)/2$$

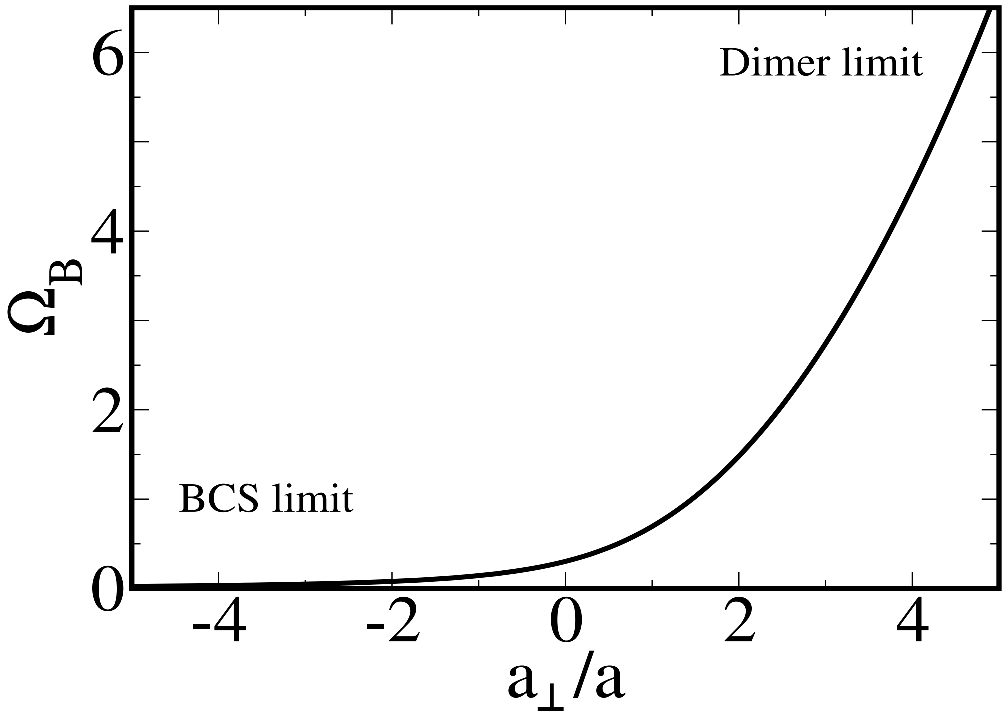
- Two-body problem has exactly one bound state (dimer) for any 3D scattering length:

$$\zeta\left(\frac{1}{2}, \Omega_B\right) = -\frac{a_\perp}{a}$$

$$\Omega_B = \frac{E_B}{2\hbar\omega_\perp}$$

$$a_\perp^2 = 2\hbar / m_0\omega_\perp$$

Dimer state: Binding energy



Recently observed experimentally *Moritz et al., PRL 2005*

Confinement induced resonance (CIR)

One open channel: Scattering solution with

1D scattering length $$a_{aa} = -\frac{a_\perp}{2}\left[\frac{a_\perp}{a} - 1.4603\right]$$

Effective 1D potential: CIR

$$V_{aa}(z,z') = g_{aa}\delta(z-z')$$

$$g_{aa} = -\frac{2\hbar^2}{m_0 a_{aa}}$$

similar to Feshbach resonance, changes sign, with diverging strength

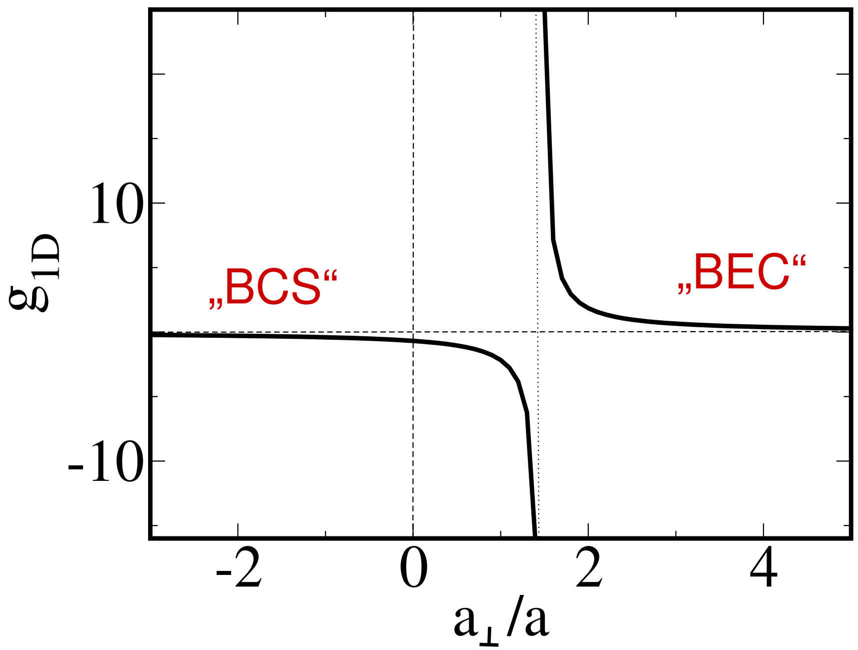
↷ 1D analogue of BCS-BEC crossover for two fermion species

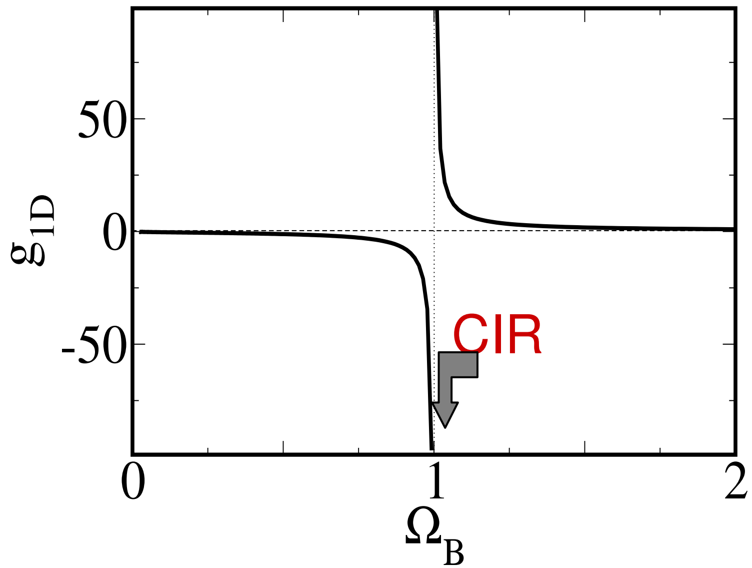
Tokatly, PRL 2004
Fuchs et al., PRL 2004

CIR in two-body interaction strength

1D atom-atom interaction g_{aa}

Olshanii, PRL 1998
Bergeman et al., PRL 2003





similar to Feshbach resonance, here due to coupling of open channel to closed channels

Low energy scattering basics in 1D

- Scattering matrix is diagonal in symmetric-antisymmetric basis

$$\psi_{in,\pm} \sim e^{ikz} \pm e^{-ikz}$$

$$S_\pm = \mp e^{i\delta_\pm(k)}$$

$$\psi_{out,\pm} = S_\pm \psi_{in,\pm}$$

- Low energy expansion needs two 1D scattering lengths

$$\delta_+(k) = -2ka_1 + O(k^2)$$

$$\delta_-(k) = +2kb_1 + O(k^2)$$

- For symmetric scattering: only a_1 matters

Atom-dimer scattering lengths

- Low energy: $E = -2\Omega_B \hbar \omega_\perp + \hbar^2 k^2 / m_0 < 0$
- Asymptotic atom-dimer scattering state

$$f(z) = e^{ikz} + h(\mathrm{sgn}(z)k, k)e^{ik|z|}$$

- Scattering amplitude expansion

$$h(k, \bar{k}) = -1 + ia_{ad}\bar{k} + ib_{ad}k + \cdots$$

Atom-dimer scattering length

Potential range parameter

- Scattering amplitude obeys integral equation derived from three-body problem

Deep dimer („BEC“) limit

$$a_\perp / a, \Omega_B \to \infty$$

Effective two-body scattering of tightly bound dimer on atom: combine 1D two-body and 3D three-body solution $a_{ad}^{3D} = 1.18a$ *Skorniakov & Ter-Martirosian, JETP 1957*

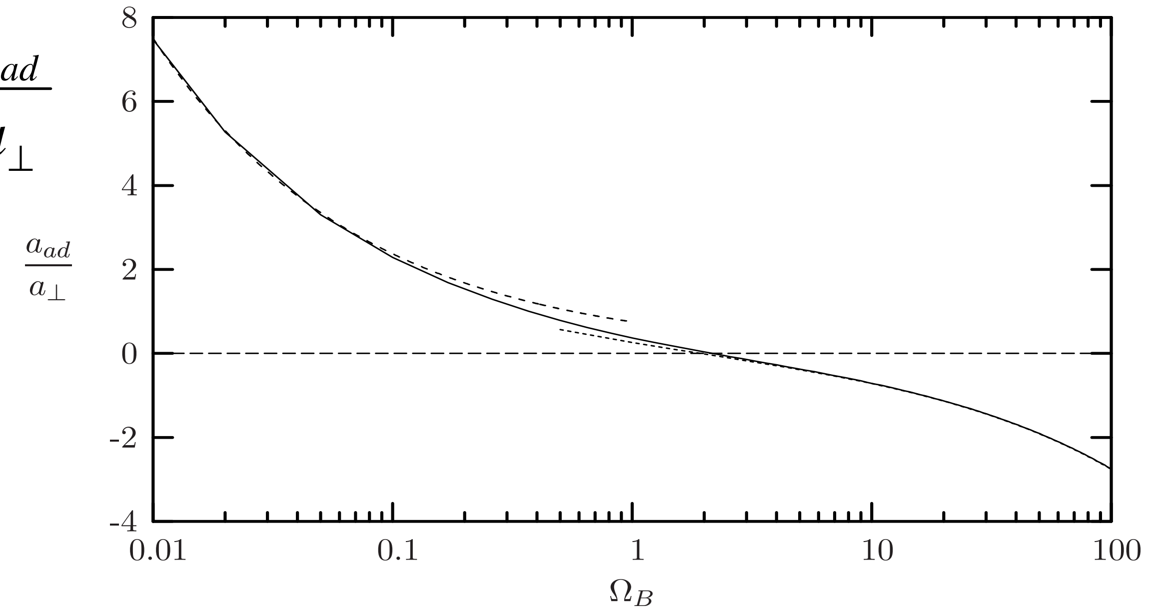
$$a_{ad} = -a_{red,\perp}^2 / 2(1.18a) \qquad a_{red,\perp}^2 = 3\hbar / 2m_0\omega_\perp$$

$$\frac{b_{ad}}{a_\perp} = \frac{8}{9}\frac{1}{\Omega_B^{3/2}} \to 0$$

1D zero-range repulsive atom-dimer interaction

Numerical crossover solution

$$\frac{a_{ad}}{a_\perp}$$

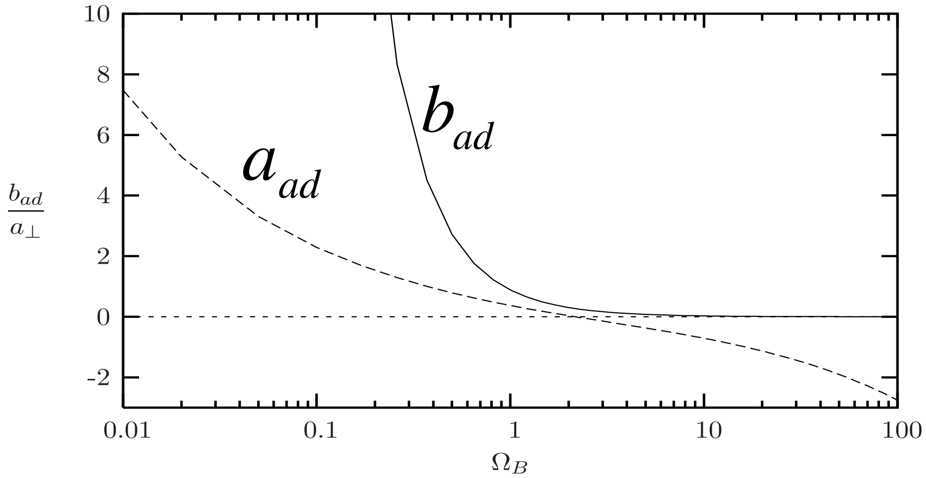


- Scattering length is zero at $\Omega_B \approx 2.2$
- BCS limit: Bethe ansatz yields exact result

$$a_{ad} = 0.75 a_\perp^2 / |a|$$

Trimer state?

Atom-dimer interaction range



Potential range diverges in BCS limit, no short-range approximation possible, but always repulsive!

No trimer state, three-body problem universal, no resonance enhancement of atom-dimer interactions!

Mora, Egger, Gogolin & Komnik, PRL 2004, PRA 2005

Dimer-dimer scattering in quasi-1D

- Dimers behave as bosons: no scattering length b_{dd}, effective δ-interaction always valid at low energy
- General form for the $|k\, a_{dd}| < 1$ scattering wavefunction:

$$f_0(z) = e^{-ik|z|} + (-1 + 2ik\, a_{dd})e^{ik|z|}$$

- Same scattering wavefunction follows for zero-range 1D potential:

$$V_{dd}(z, z') = g_{dd}\,\delta(z - z') \qquad g_{dd} = -\frac{2\hbar^2}{(2m_0)a_{dd}}$$

Our aim: calculate a_{dd} as function of $a_\perp / a$

Mora, Komnik, Egger & Gogolin, PRL 2005

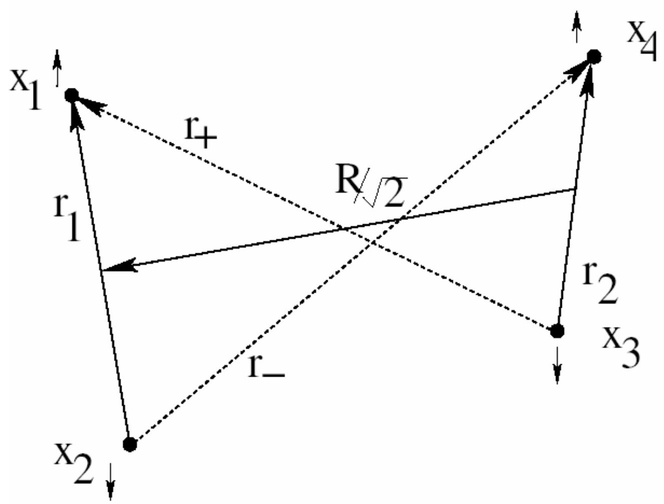
Four-body (↑↑↓↓) problem

$$\left[-\left(\Delta_{r_1}+\Delta_{r_2}+\Delta_R\right)+U_c(\vec{r}_1)+U_c(\vec{r}_2)+U_c(\vec{R})+\sum_{\pm,1,2}V(r_\pm)\right]\Psi=E\Psi$$

- Pseudopotential implies Bethe-Peierls boundary conditions

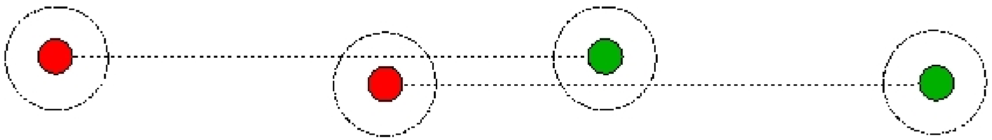
$$\Psi\left(\vec{r}_1\to 0,\vec{r}_2,\vec{R}\right)=\frac{f\left(\vec{r}_2,\vec{R}\right)}{4\pi r_1}\left(1-{r_1}/{a}\right)$$

$$f\left(\vec{r}_2,\vec{R}\right)=f\left(-\vec{r}_2,-\vec{R}\right)$$



- Integral equation for $f\left(\vec{r}_2,\vec{R}\right)$

BCS limit: reflectionless scattering



- Bethe ansatz solves problem exactly: wavefunction for far apart dimers describes reflectionless scattering

$$f(z) = \cos kz - (1 + ik\, a_{dd})^{-1} e^{ik|z|}$$

$$a_{dd} = a_{aa} / 2$$

- Exactly same scattering amplitude as δ-interaction for arbitrary k
- Dimers not broken during scattering: composite nature of bosons not apparent
- Bound state must be projected away, no tetramer state

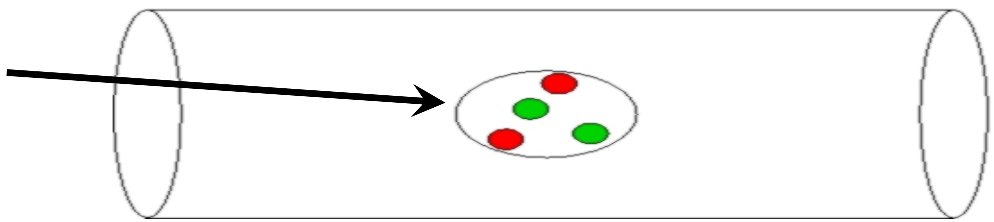
BEC limit: Exact result

Again use 1D two-body result & establish contact to dimer-dimer scattering solution in 3D

$$a_{dd}^{3D} = 0.6a$$

Petrov et al., PRL 2004

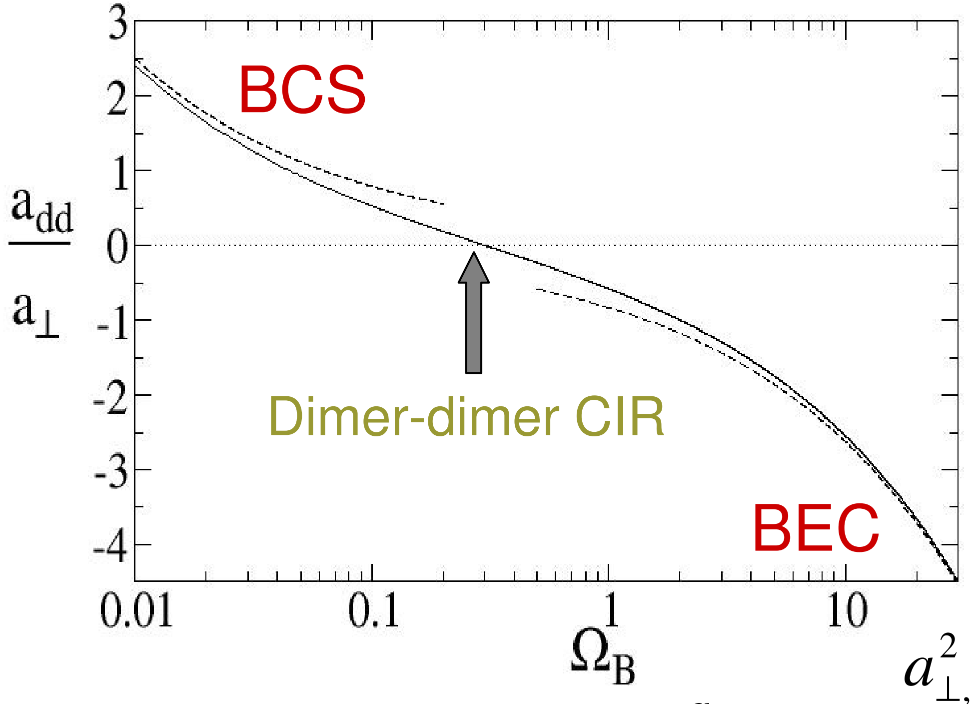
$$a_{dd} = -\frac{a_{\perp,red}^2}{2(0.6a)}$$



Crossover regime: solve integral equation

Mora, Komnik, Egger & Gogolin, PRL 2005

$$a_{dd} = -\frac{1}{4}\frac{a_\perp^2}{a} = \frac{a_{aa}}{2}$$



$$a_{dd} = -\frac{a_{\perp,red}^2}{2(0.6a)}$$

CIR in dimer-dimer scattering at

$$\Omega_B \approx 0.3$$

Many-body solution

Tokatly, PRL 2004
Fuchs et al. PRL 2004
Mora et al. PRL 2005

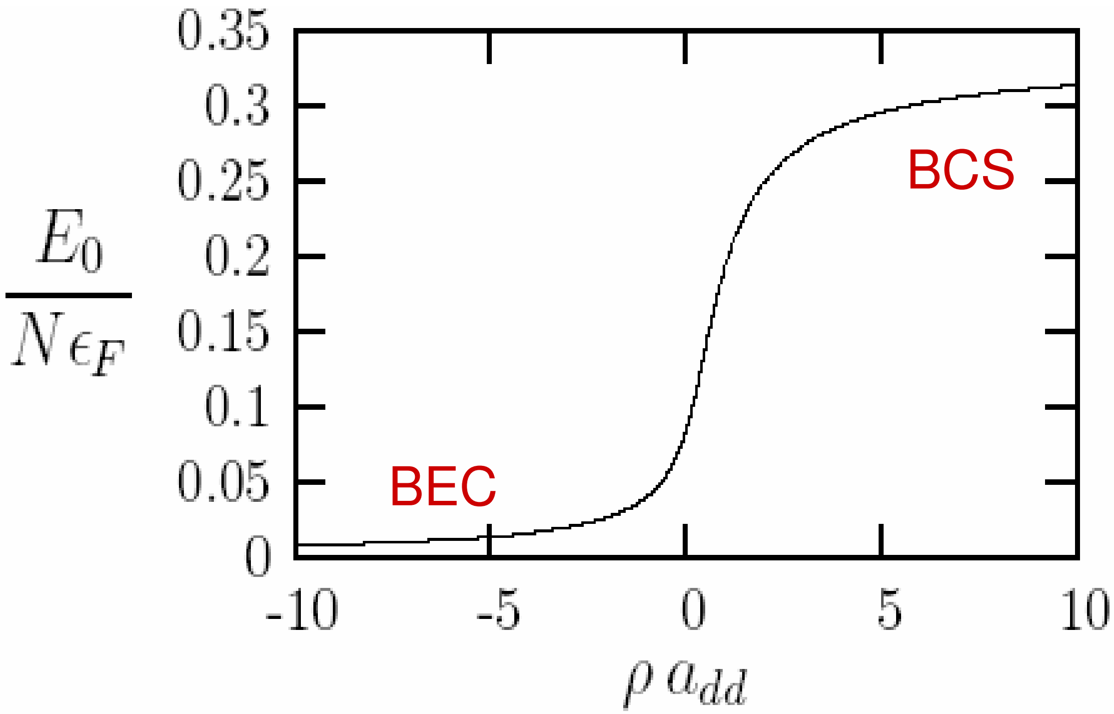
- *N* fermions reduced to *N/2* bosons (dimers) with delta-interactions for $\rho a_\perp < 1$ parametrized by a_{dd}
- Bound state for attractive dimer-dimer interaction (BCS limit) must be projected away
- Full many-body crossover for ground state is exactly solvable: Lieb-Liniger equations

$$\frac{E_0}{N} = -\hbar\omega_\perp\Omega_B + \frac{1}{\rho}\int_{-K}^{K} dk\, \frac{\hbar^2 k^2}{4m_0} f(k), \qquad \int_{-K}^{K} dk\, f(k) = \rho/2$$

$$2\pi\, f(k) = 1 - \frac{4}{a_{dd}} \int_{-K}^{K} dk\, \frac{f(p)}{4/a_{dd}^2 + (p-k)^2}$$

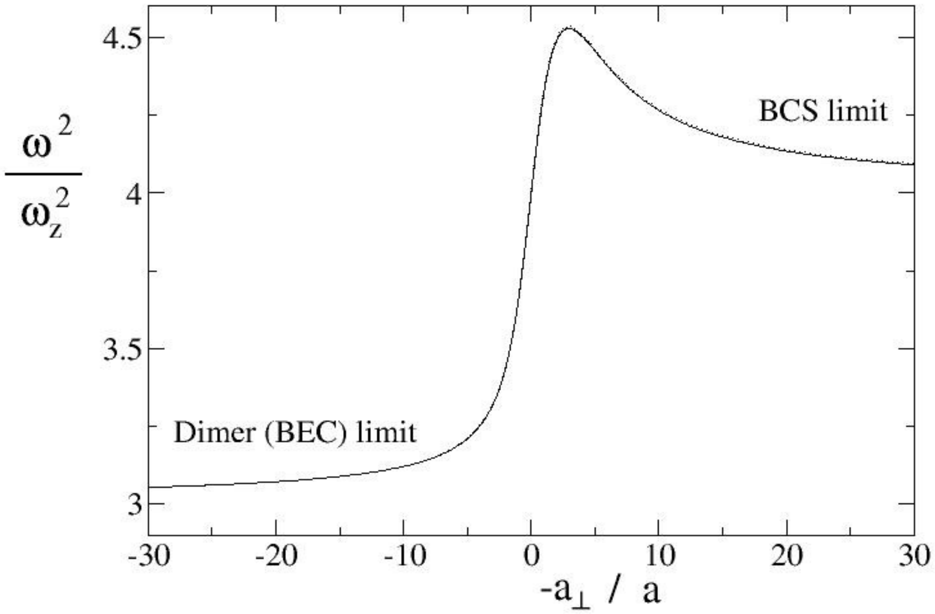
- BCS limit: reproduces Yang-Gaudin equations

Ground-state energy

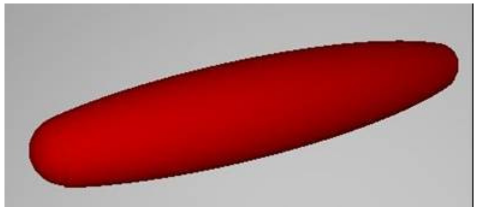


Breathing mode oscillation frequency

For fermions in elongated trap:



$\omega_z << \omega_\perp$



What about bosonic atoms?

Mora, Egger & Gogolin, PRA 2005

- Three-boson problem can be analyzed along the same lines, but requires short-distance regularization in pseudopotential approach
- Bound trimer states:
  - Efimov trimer states (away from „BCS“ limit)
  - A single confinement-induced trimer state in quasi-1D traps is predicted for arbitrary $a_\perp / a$
  - Interplay with Efimov states only understood in dimer limit so far…

Disordered interacting bosons

- Field theory unstable for bosons
- So far only mean-field type approximate results, or numerical simulations
- Exact statements possible for 1D disordered bosons with strong repulsion:
  - Bose-Fermi mapping to free disordered fermions
  - Bose glass phase is mapped to Anderson localized fermionic phase

De Martino, Thorwart, Egger & Graham, PRL 2005

Bose Hubbard model

- Bose Hubbard model in 1D

$$H = \sum_l \left(-J\left(b_{l+1}^* b_l + h.c.\right) + (h_l + bl^2)n_l + Un_l\left(n_l - 1\right)\right)$$

- Tunable on-site disorder $\langle h_l h_k \rangle_{dis} = \Delta_{dis}\delta_{lk}$
  - laser speckle pattern
  - incommensurate additional lattice
  - microchip-confined systems: Atom-surface interactions

Bose-Fermi Mapping

Consider hard-core bosons: $U \to \infty$

↷ only $n_l = 0,1$ possible!

Jordan-Wigner transformation to free fermions:

$$b_l = \exp\left(i\pi \sum_{j<l} c_j^* c_j \right) c_l$$

↷

$$H_F = \sum_l \left(-J\left(c_{l+1}^* c_l + h.c. \right) + (h_l + bl^2) c_l^* c_l \right)$$

Well known in clean case (Tonks-Girardeau), but also works with disorder!

Many-body boson wavefunction

N-boson wavefunction is Slater determinant of free fermion solutions $\psi_i(l)$ to single-particle energy ε_i

$$\Phi_\nu^B(l_1,\cdots,l_N)=\frac{1}{\sqrt{N!}}\left|\det\psi_i(l_j)\right|$$

$$E_\nu = \sum_{j=1}^{N} \varepsilon_i^{(j)}$$

Girardeau, J. Math. Phys. 1960

Physical observables

- All observables expressed by $\left|\Phi_\nu^B\right|$ are invariant under Bose-Fermi mapping, e.g. local density of states (LDoS)

$$\rho(\varepsilon, l) = \sum_{\nu} \sum_{l_2,\ldots,l_N} \delta(\varepsilon - E_\nu) \left|\Phi_\nu^B(l, l_2, \ldots, l_N)\right|^2$$

- Greatly simplified calculation for others, e.g. boson momentum distribution

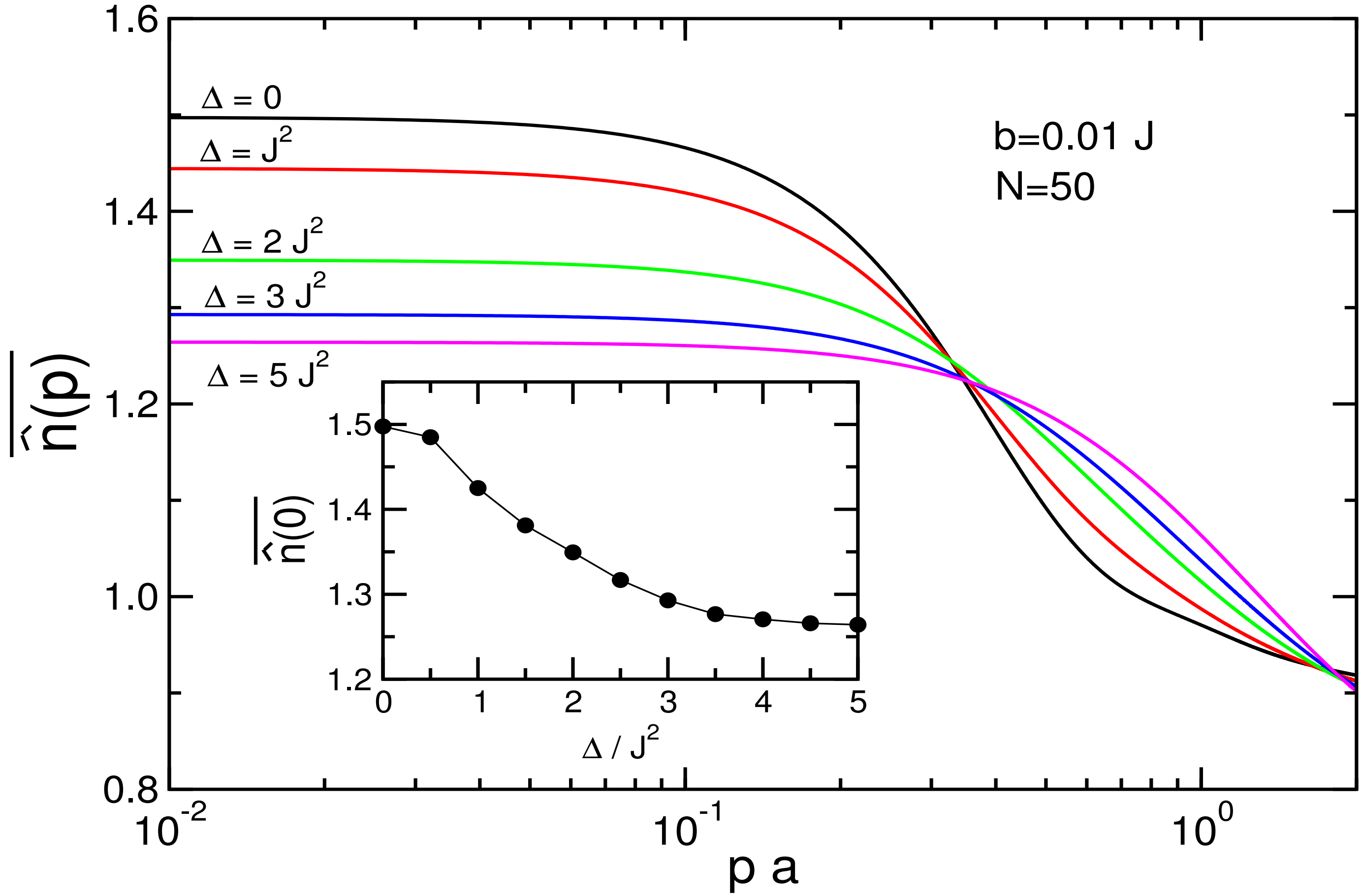
Boson momentum distribution

- Momentum distribution different for boson and fermion systems
- Bosonic one: $\hat{n}(p) = \frac{1}{N} \sum_{ll'} e^{-ip(l-l')a} \langle b_l^* b_{l'} \rangle$
- Jordan-Wigner transformation & Wick´s theorem give for fixed disorder:

$$\langle b_l^* b_{l'} \rangle = 2^{l-l'-1} \det G^{(l,l')}$$

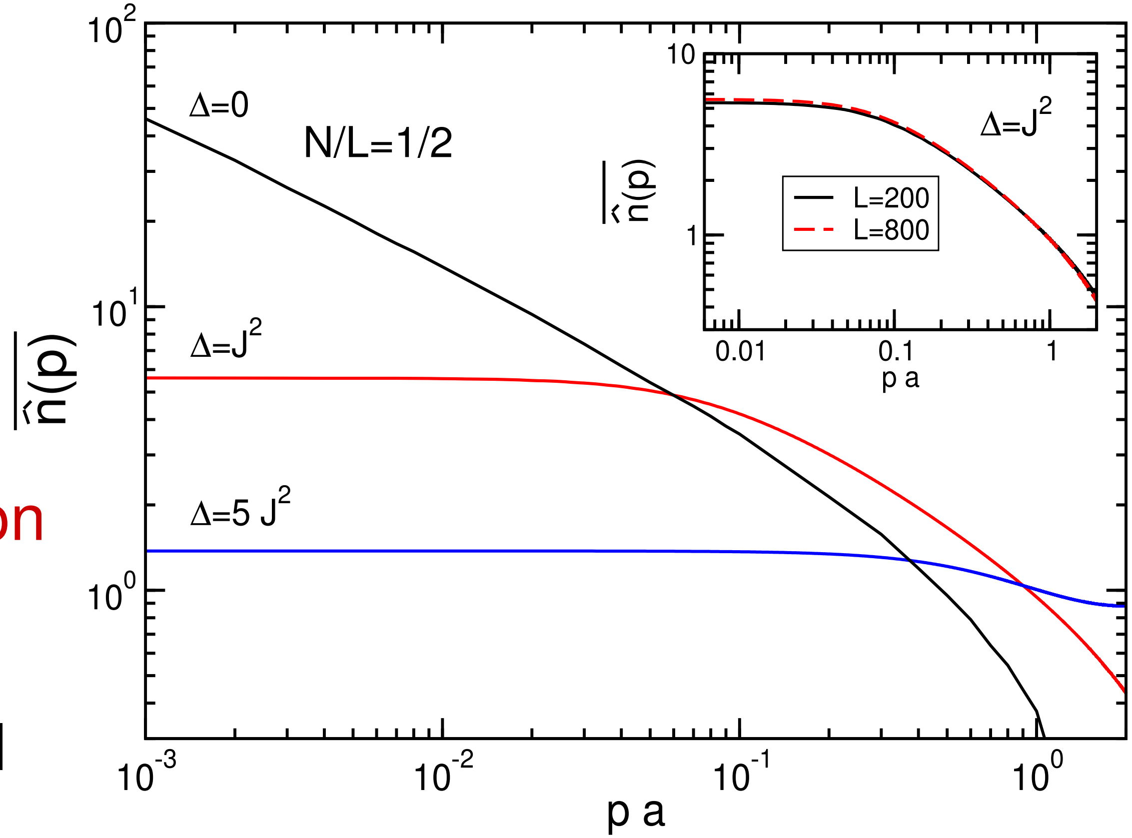
$$G_{ij}^{(l,l')} = \langle c_{l'+i}^* c_{l'+j-1} \rangle - \frac{1}{2} \delta_{i,j-1}$$

Re



Numerically averaged over 300 disorder realizations, T=0

Momentum distribution on a ring



Complete destruction of quasi-long range order by disorder apparent from small *p* behavior

Continuum limit (homogeneous case)

- Low-energy expansion defines bispinor

$$c_l \approx \sqrt{a}\left[e^{ik_F x}\Psi_R(x) + e^{-ik_F x}\Psi_L(x)\right] \qquad x = la$$

- Free-fermion Hamiltonian $H = \int dx \Psi^* \hat{h} \Psi$

$$\hat{h} = -iv_F \sigma_z \partial_x + \mu(x) + \xi(x)\sigma_+ + \xi^*(x)\sigma_-$$

with $k_F = \pi N / L,\ v_F = k_F / m$

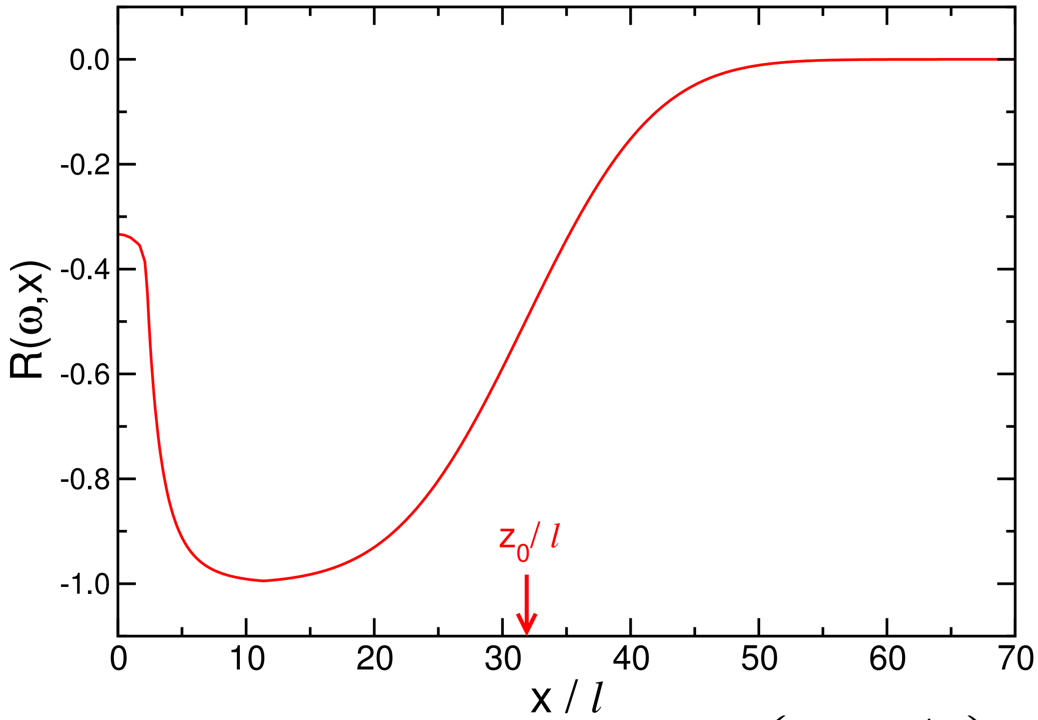
Disorder averages

- Disorder forward scattering can be eliminated by gauge transformation for incommensurate situation
- Backward scattering: $\left\langle \xi(x)\xi^*(x')\right\rangle_{dis} = \frac{v_F^2}{2\ell}\delta\left(x-x'\right)$
- Consider weak disorder: $k_F \ell > 1$
- Standard free-fermion Hamiltonian for study of 1D Anderson localization, many results available (mainly via Berezinskii method)

Spectral correlations

- LDoS correlations at different energies and locations $$R(\Omega, x-x') = \langle \tilde{\rho}(\varepsilon, x)\tilde{\rho}(\varepsilon+\Omega, x')\rangle - 1$$
  - equals the fermionic correlator
  - consider low energies $\Omega < v_F / \ell$
- Limits: *Gorkov, Dorokhov & Prigara, Zh.Eksp.Teor.Fis.1983*
  - Large distances: uncorrelated value *R=0*
  - Short distance: *R* approaches *-1/3*
  - Deep dip at intermediate distances

Spectral fluctuations



Deep dip for $\ell < |x - x'| < z_0 = 2\ell \ln\left({}^{8v_F}\!/_{\Omega\ell} \right)$

Then: $R(\Omega, x) = \frac{1}{2}\left[erf\left(\frac{x - z_0}{2\sqrt{x z_0}} \right) - 1 \right]$

Implications

- Energetically close-by states occupy
  - with high probability distant locations
  - but appreciable overlap at short distances
- Localized states are centered on many defects, complicated quantum interference phenomenon
- No Wigner-Dyson correlations, but Poisson statistics of uncorrelated energy levels

Other quantities…

Mapping allows to extract many other experimentally relevant quantities:

- Compressibility, and hence sound velocity
- Density-density correlations, structure factor
- Distribution function of LDoS
- Time-dependent density profile (expansion)
  - crossover from short-time diffusion to long-time localization physics

De Martino, Thorwart, Egger & Graham, PRL 2005

Conclusions

Many body effects are very pronounced in quasi-1D cold atom systems

- Few-body and many-body scattering problem can be solved in confined geometry, 1D analogue to BEC-BCS crossover
- Strongly interacting 1D bosons: Mapping to noninteracting fermions allows to apply solution of 1D Anderson localization