Multichannel Kondo dynamics and Surface Code from Majorana bound states

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Dresden workshop
14-18 Sept. 2015
Overview

- Brief introduction to Majorana bound states

- Majorana-Cooper box: a Majorana-based „quantum impurity spin“

- ‚Topological‘ Kondo effect: a single box connected to normal leads → stable non-Fermi liquid fixed point of multi-channel Kondo type
  Beri & Cooper, PRL 2012
  Altland & Egger, PRL 2013
  Altland, Beri, Egger & Tsvelik, PRL 2014

- 2D array of boxes: towards realistic implementations of Majorana surface codes
  Landau, Plugge, Sela, Altland, Albrecht & Egger, preprint;
  Terhal, Hassler & DiVincenzo, PRL 2012;
  Vijay, Hsieh & Fu, arXiv:1504.01724
Majorana bound states

- Majorana fermion is its own antiparticle $\gamma = \gamma^+$
  - carries no charge
  - real-valued solution of relativistic Dirac equation
- Majorana bound state (MBS): a localized zero mode excitation
  - Condensed matter realizations: equal weight superposition of electron and hole states in superconductors
Majorana algebra

Consider set of Majorana bound states at different locations in space

Self-adjoint operators \( \gamma_j = \gamma_j^+ \)

Clifford algebra \( \gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij} \)

- Different Majorana operators anticommute just like fermions
- But: \( \gamma_j^+ \gamma_j = \gamma_j^2 = 1 \)
  - annihilation of particle & antiparticle recovers previous state
  - Occupation number of single MBS is ill-defined
Counting Majoranas

count states of a Majorana pair via non-local auxiliary fermion occupation number

c = (γ₁ + iγ₂)/2

γ₁ = c + c⁺

γ₂ = −i(c − c⁺)

iγ₁γ₂ = 2c⁺c − 1

c⁺c = (iγ₁γ₂ + 1)/2 = 0,1

MBS = „half a fermion“, fractionalized zero mode
MBS in p-wave superconductors

- Bogoliubov quasiparticles in s-wave BCS superconductors? \( \gamma = uc^+ + vc_\downarrow \neq \gamma^+ \)
  - spin spoils it: no MBS possible!

- better: spinless quasiparticles in p-wave superconductor
  - Energy at Fermi level: \( \gamma = uc^+ + u^*c = \gamma^+ \)
  - Vortex in 2D p-wave superconductor hosts MBS

- Experimentally most promising route (at present):
  MBS as end states of 1D p-wave superconductors:
  Kitaev chain
Realizing the Kitaev chain

InAs or InSb helical nanowires host Majoranas due to interplay of
• strong Rashba spin orbit field
• magnetic Zeeman field
• proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

Transport signature of Majoranas:
Zero-bias conductance peak due to resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009
Flensberg, PRB 2010

Copenhagen group (new results)
Majorana-Cooper box

**N helical nanowires** proximitized by same mesoscopic floating superconductor

→ Coulomb charging energy important

On energy scales below proximity gap:

\[
H_{Box} = E_C \left( 2N_c + \sum_{\alpha=1}^{N} c_{\alpha}^+ c_{\alpha} - n_g \right)^2
\]

- 2N Majorana end states (at E=0 for long wires)
  → N fermionic zero modes \( c_\alpha \)
- Condensate gives bosonic zero mode
  Cooper pair number \( N_c \), conjugate supercond. phase \( \phi \)

*Fu, PRL 2010
Hützen et al., PRL 2012
Beri & Cooper, PRL 2012
Quantum „impurity spin“

For near integer gate parameter: Uniqueness of equilibrium charge state implies total parity constraint

\[ Q = 2N_c + \sum_{\alpha=1}^{N} c_\alpha^+ c_\alpha = \text{integer const} \]

\[ i^{N} \prod_{j=1}^{2N} \gamma_j = \pm 1 \]

\[ c_\alpha = \left( \gamma_{2\alpha-1} + i \gamma_{2\alpha} \right) / 2 \]

- Degeneracy of Majorana sector = \(2^{N-1}\)
  - Parity constraint removes half the states
- For \(N>1\) „quantum impurity spin“ nonlocally encoded by Majorana bound states on box
Topological Kondo effect

- Couple „impurity spin“ to normal leads (e.g. overhanging helical nanowire parts): Cotunneling causes „exchange coupling“
- Robust non-Fermi liquid multi-channel Kondo fixed point
- Observable in electric conductance or shot noise measurements

Beri & Cooper, PRL 2012
Altland & Egger, PRL 2013; Beri, PRL 2013
Altland, Beri, Egger & Tsvelik, PRL 2014
Normal leads

1D helical Dirac fermions describe the normal wires (lead $j=1,\ldots,M$)

- Semi-infinite leads, tunnel-coupled individually to Majorana states at $x=0$
- Pair of right/left movers for $x>0$, with $\psi_{j,L}(0) = \psi_{j,R}(0)$

„Unfolded“ Hamiltonian

$$\psi_L(x) = \psi_R(-x)$$

$$H_{leads} = -iv_F \sum_{j=1}^{M} \int_{-\infty}^{\infty} dx \, \psi_j^+ \partial_x \psi_j$$
Tunneling Hamiltonian

\[ H_t = \sum_j t_j \psi_j^+(0) e^{-i\phi/2} \gamma_j + \text{h.c.} \]

- Respect charge conservation (floating device)
- Spin structure of Majorana states encoded in tunnel matrix elements
- Next step: Schrieffer-Wolff transformation to project onto degenerate ground state of box

Beri & Cooper, PRL 2012
Flensberg, PRB 2010
Fu, PRL 2010
Zazunov, Levy Yeyati & Egger, PRB 2011
Topological Kondo effect

\[ H = -i v_F \int_{-\infty}^{\infty} dx \sum_{j=1}^{M} \psi_j^+ \partial_x \psi_j + i \lambda \sum_{j \neq k} \psi_j^+(0) S_{jk} \psi_k(0) \]

\[ \lambda \approx \frac{t^2}{E_C} \]

- Majorana bilinears \( S_{jk} = i \gamma_j \gamma_k \)

- Majorana 'reality' condition: „quantum impurity spin“ obeys \( \text{SO}(M) \) algebra [instead of \( \text{SU}(2) \)]

- Nonlocality ensures stability of Kondo fixed point: deviations from isotropy are RG irrelevant
Example: Minimal case $M=3$

allows for spin-1/2 representation

\[
S_x = \frac{i}{4} \gamma_2 \gamma_3, \quad S_y = \frac{i}{4} \gamma_3 \gamma_1, \quad S_z = \frac{i}{4} \gamma_1 \gamma_2
\]

\[
\left[ S_x, S_y \right] = iS_z
\]

- can be represented by standard Pauli matrices
- "spin" exchange-coupled to effective spin-1 lead

→ overscreened multi-channel Kondo effect

Residual ground state degeneracy
local non-Fermi liquid behavior
Linear conductance tensor

\[ G_{jk} = e \frac{\partial I_j}{\partial \mu_k} = \frac{2e^2}{h} \left( 1 - \left( \frac{T}{T_K} \right)^{2y-2} \right) \left[ \delta_{jk} - \frac{1}{M} \right] \]

asymptotic low-temperature behavior

- Non-integer scaling dimension \( y = 2 \left( 1 - \frac{1}{M} \right) > 1 \)
- implies non-Fermi liquid behavior
- completely isotropic multi-terminal junction
Majorana surface code

- Network of interacting Majorana fermions → realization of Majorana surface code

- Surface code quantum computation
  - Encode 'logical' qubit by entangling many physical qubits
  - Comparatively simple 2D array layouts
  - Error tolerance orders of magnitude better than in alternative approaches
  - Error detection without need for active error correction & controlled by classical software
  Review: Fowler, Mariantoni, Martinis & Clarke, PRA 2012
Scalability issues

- Reasonably fault-tolerant logical qubit needs $>10^3$ physical qubits $\rightarrow$ we need about $10^8$ physical qubits to factorize 100-digit integer

- **Maximal simplicity** in implementation and access to physical qubits required

- Semiconductor Majorana layouts may offer this simplicity

  - Single-step readout without ancilla qubits possible
    
    *Vijay, Hsieh & Fu, arXiv:1504.01724*

  - Qubit readout & manipulation through tunnel probes & SETs, without radiation fields or flux interferometry
    
    *Landau, Plugge, Sela, Altland, Albrecht & Egger, preprint*
Blueprint: 2D array of boxes

(c) \[ t_{ll'} \]

(l) \[ l' \]

(b) \[ a = 2 \]
\[ \gamma_1^\alpha \]
\[ \gamma_2^\alpha \]
\[ a = 1 \]

Majorana-Cooper box

(a)
2D array of Majorana-Cooper boxes

- Building block: Majorana-Cooper box with two proximitized helical nanowires
  - joined by superconductor slab: finite common charging energy
  - each box hosts M=4 Majorana zero modes
    - All wires in array parallel: homogeneous Zeeman field
      → simultaneous topological transition

- Now couple neighboring boxes by tunnel bridges

\[ H_t = -\frac{t_{ll'}}{2} \gamma_l \gamma_{l'} e^{i(\varphi_l - \varphi_{l'})/2} + \text{h.c.} \]
Stabilizers: plaquette operators

- Low-energy excitations of array correspond to minimal loop structures
  - Minimal loop contains 8 Majorana operators
- Hermitian plaquette operator for loop no. \( n \)
  \[ O_n = \prod_{j=1}^{8} \gamma_j^{(n)} \]
- Set of mutually commuting operators
  - Plaquette eigenvalues = \( \pm 1 \): simultaneously measurable set of physical qubits
  - Serve as stabilizers of the surface code
Plaquette Hamiltonian

- Schrieffer-Wolff transformation $\rightarrow$ low-energy theory

\[ H_{\text{code}} = -\sum_{n} \text{Re}(c_n) O_n \]

\[ c_n = \frac{5}{16} \prod \frac{t_{l_n l_n'}}{E_C^3} \]

- Amplitude $c_n$ for n-th loop contains product of four tunnel amplitudes along loop
- Surface code works although the Re($c_n$) are generally uncorrelated random energies

- How to measure and manipulate plaquettes?
  - Essential ingredient for surface code quantum information processing
Surface code operation principles

- Permanent repetition of sequential stabilizer measurements
  - Project code onto eigenstate of stabilizer system
  - Occasional erroneous plaquette flips are simply recorded (& corrected by classical software), no active error correction needed
  - Punching „holes“ by ceasing measurements at plaquette(s): binary eigenstates of Wilson loops around holes serve as logical qubits

- need maximally simple readout & controlled flip of stabilizers

Fowler et al., PRA 2012
Attaching tunnel probes and SETs

Read out of stabilizers: tunnel conductance via attached leads, noninvasive (no plaquette flip) measurement

Controlled plaquette flip: Transfer 1 electron through code by changing gate voltages on attached pair of SETs
Tunnel conductance

Attach pair of tunnel contacts (normal leads)

\[ H_{tunn} = \sum_{j=1,2} \lambda_j \psi_j^+(0) \ e^{-i\varphi_j/2} \gamma_j + \text{h.c.} \]

\( \gamma_j \) anticommutes with the two \( O_n \) containing \( \gamma_j \)
... and commutes with all other \( O_n \)

Neighboring \( \gamma_{1,2} \) belong to same \( O_n \) pair
→ double flip, i.e., all plaquettes remain invariant
→ this specific conductance measurement is noninvasive
Quantitative results for conductance

Schrieffer-Wolff transformation with leads:
effective coupling Hamiltonian

\[ H_{\text{eff}} = \alpha \left( \xi + c_A^* O_A + c_B^* O_B \right) \psi_1^+(0) \psi_2(0) + \text{h.c.} \]

\[ \alpha = -\frac{32}{5} \frac{\lambda_1 \lambda_2^*}{t_{12} E_C} \]

\[ c_n = \frac{5}{16} \frac{\prod t_{l_n l_n'}}{E_C^3} \]

- two paths around plaquettes A & B
- "direct" amplitude \( \xi \) from \( 1 \rightarrow 2 \) vanishes for integer \( n_g \) (but finite away from valley center)
- Tunnel conductance from lead \( 1 \rightarrow 2 \) follows from perturbation theory
Tunnel conductance

Interference terms: tunnel conductance is sensitive to A and B plaquette eigenvalues

\[ G_{12} \propto |\alpha|^2 \left( g_0 + g_A O_A + g_B O_B + g_{AB} O_A O_B \right) \]
Manipulation: flipping plaquettes

- Use pair of SETs to adiabatically pump single electron through array, tunneling in (out) at $\gamma$ ($\gamma'$). 
- Change SET configuration $(1,0) \rightarrow (0,1)$ via gate voltages.
- Arbitrary Majorana pair has 0, 1, or 2 plaquettes in common.
- This electron transfer flips 4, 2, or 0 plaquettes.
  - No plaquette flipped: recover non-invasive case.
  - Minimal excitation: 2 flipped plaquettes, cf. figure.
- Activation of SET pairs at arbitrary places: create and move excitations arbitrarily.

Flensberg, PRL 2011
Towards hardware layout
Conclusions

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THANK YOU FOR YOUR ATTENTION!