Multichannel Kondo dynamics and Surface Code from Majorana bound states

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Overview

- Brief introduction to Majorana bound states

  Alicea, Rep. Prog. Phys. 2012

- Majorana-Cooper box: a Majorana-based „quantum impurity spin“
- ‚Topological‘ Kondo effect: a single box connected to normal leads → stable non-Fermi liquid fixed point of multi-channel Kondo type

  Beri & Cooper, PRL 2012
  Altland & Egger, PRL 2013
  Altland, Beri, Egger & Tsvelik, PRL 2014

- 2D array of boxes: towards realistic implementations of Majorana surface codes

  Landau, Plugge, Sela, Altland, Albrecht & Egger, preprint;
  Terhal, Hassler & DiVincenzo, PRL 2012;
  Vijay, Hsieh & Fu, arXiv:1504.01724

Majorana bound states

- Majorana fermion is its own antiparticle $\gamma = \gamma^{+}$
  - carries no charge
  - real-valued solution of relativistic Dirac equation
- Majorana bound state (MBS): a localized zero mode excitation
  - Condensed matter realizations: equal weight superposition of electron and hole states in superconductors

Majorana algebra

Consider set of Majorana bound states

at different locations in space

Self-adjoint operators $\gamma_j = \gamma_j^+$

Clifford algebra $\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij}$

- Different Majorana operators anticommute just like fermions
- But: $\gamma_j^+ \gamma_j = \gamma_j^2 = 1$
  - annihilation of particle & antiparticle recovers previous state
  - Occupation number of single MBS is ill-defined

Counting Majoranas

count states of a Majorana pair via non-local auxiliary fermion occupation number

$$c = (\gamma_1 + i\gamma_2)/2$$

$$c^+ c = (i\gamma_1\gamma_2 + 1)/2 = 0,1$$

$$\gamma_1 = c + c^+$$

$$\gamma_2 = -i(c - c^+)$$

MBS = „half a fermion“,
fractionalized zero mode

$$i\gamma_1\gamma_2 = 2c^+ c - 1$$

MBS in p-wave superconductors

- Bogoliubov quasiparticles in s-wave BCS superconductors? $\gamma = uc_\uparrow^+ + vc_\downarrow \neq \gamma^+$

  spin spoils it: no MBS possible!

- better: spinless quasiparticles in p-wave superconductor
  - Energy at Fermi level: $\gamma = uc^+ + u^* c = \gamma^+$
  - Vortex in 2D p-wave superconductor hosts MBS
  - Experimentally most promising route (at present):

MBS as end states of 1D p-wave superconductors: Kitaev chain

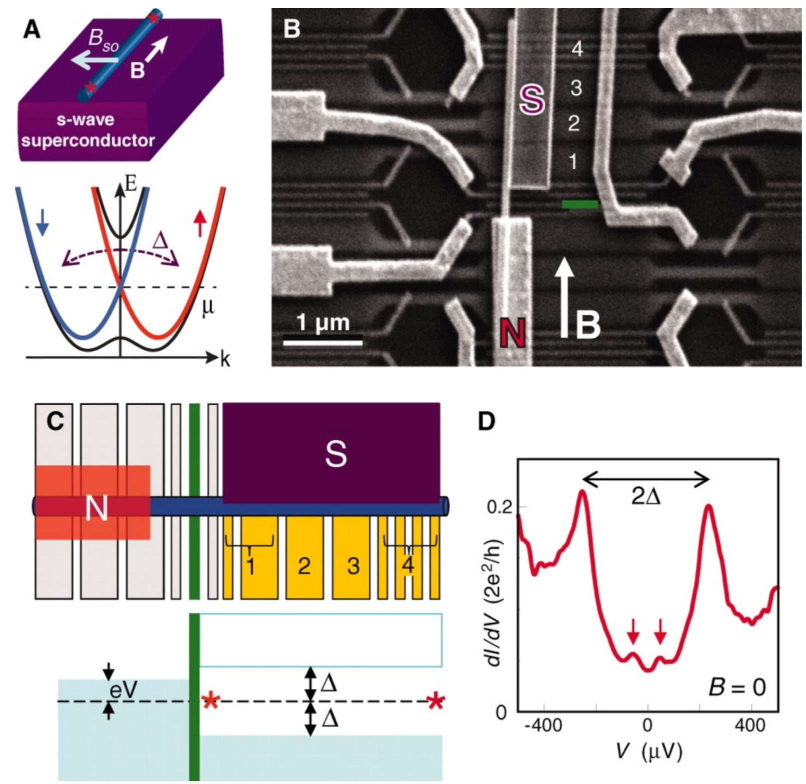
Realizing the Kitaev chain

InAs or InSb helical nanowires host Majoranas due to interplay of

- strong Rashba spin orbit field
- magnetic Zeeman field
- proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

Mourik et al., Science 2012



Transport signature of Majoranas: Zero-bias conductance peak due to resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009
Flensberg, PRB 2010

see also*: Rokhinson et al., Nat. Phys. 2012; Deng et al., Nano Lett. 2012; Das et al., Nat. Phys. 2012; Churchill et al., PRB 2013; Nadj-Perge et al., Science 2014; Copenhagen group (new results)*

Majorana-Cooper box

Fu, PRL 2010
Hützen et al., PRL 2012
Beri & Cooper, PRL 2012

N helical nanowires proximitized by same mesoscopic floating superconductor

→ Coulomb charging energy important

On energy scales below proximity gap:

$$H_{Box} = E_C \left(2N_c + \sum_{\alpha=1}^{N} c_\alpha^+ c_\alpha - n_g \right)^2$$

gate parameter

- 2N Majorana end states (at E=0 for long wires)
  → N fermionic zero modes c_α
- Condensate gives bosonic zero mode
  Cooper pair number N_c, conjugate supercond. phase φ

Quantum „impurity spin“

For near integer gate parameter: Uniqueness of equilibrium charge state implies total parity constraint

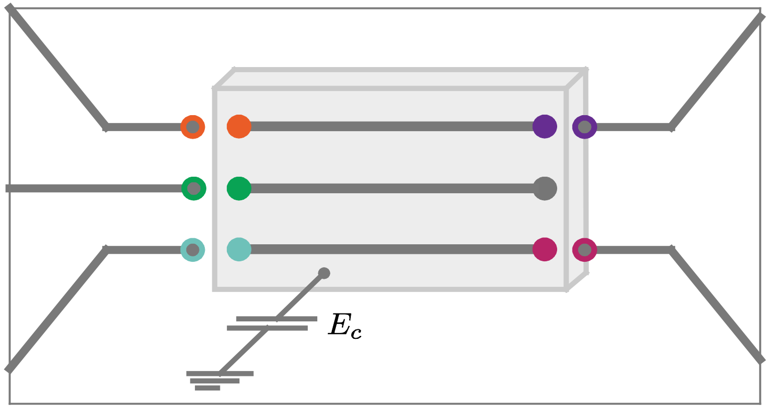
$$Q = 2N_c + \sum_{\alpha=1}^{N} c_\alpha^+ c_\alpha = \text{integer const}$$

$$\Longrightarrow \quad i^N \prod_{j=1}^{2N} \gamma_j = \pm 1 \qquad c_\alpha = \left(\gamma_{2\alpha-1} + i\gamma_{2\alpha}\right)/2$$

- Degeneracy of Majorana sector = 2^{N-1}

  parity constraint removes half the states
- For N>1 „quantum impurity spin“ nonlocally encoded by Majorana bound states on box

Topological Kondo effect



Beri & Cooper, PRL 2012
Altland & Egger, PRL 2013; Beri, PRL 2013
Altland, Beri, Egger & Tsvelik, PRL 2014
Zazunov, Altland & Egger, New J. Phys. 2014

- Couple „impurity spin“ to normal leads (e.g. overhanging helical nanowire parts): Cotunneling causes „exchange coupling“
- Robust non-Fermi liquid multi-channel Kondo fixed point
- observable in electric conductance or shot noise measurements

Normal leads

1D helical Dirac fermions describe the normal wires (lead j=1,...,M)

- Semi-infinite leads, tunnel-coupled individually to Majorana states at x=0
- Pair of right/left movers for x>0, with $\psi_{j,L}(0)=\psi_{j,R}(0)$

„Unfolded" Hamiltonian $\psi_L(x)=\psi_R(-x)$

$$H_{leads} = -iv_F \sum_{j=1}^{M} \int_{-\infty}^{\infty} dx \; \psi_j^+ \partial_x \psi_j$$

Tunneling Hamiltonian

Flensberg, PRB 2010
Fu, PRL 2010
Zazunov, Levy Yeyati & Egger, PRB 2011

$$H_t = \sum_j t_j \psi_j^+(0)\, e^{-i\varphi/2} \gamma_j + \text{h.c.}$$

- Respect charge conservation (floating device)
- Spin structure of Majorana states encoded in tunnel matrix elements
- Next step: Schrieffer-Wolff transformation to project onto degenerate ground state of box

Beri & Cooper, PRL 2012

Topological Kondo effect

$$H = -iv_F \int_{-\infty}^{\infty} dx \sum_{j=1}^{M} \psi_j^+ \partial_x \psi_j + i\lambda \sum_{j\neq k} \psi_j^+(0) S_{jk} \psi_k(0)$$

$$\lambda \approx \frac{t^2}{E_C}$$

Majorana bilinears $S_{jk} = i\gamma_j \gamma_k$

- Majorana ‚reality' condition: „quantum impurity spin" obeys SO(M) algebra [instead of SU(2)]
- Nonlocality ensures stability of Kondo fixed point: deviations from isotropy are RG irrelevant

Example: Minimal case M=3

allows for spin-1/2 representation

$$S_x = \frac{i}{4}\gamma_2\gamma_3 \qquad S_y = \frac{i}{4}\gamma_3\gamma_1 \qquad S_z = \frac{i}{4}\gamma_1\gamma_2$$

$$\left[S_x, S_y\right]_- = iS_z$$

- can be represented by standard Pauli matrices
- „spin“ exchange-coupled to effective spin-1 lead

→ overscreened multi-channel Kondo effect

Residual ground state degeneracy

local non-Fermi liquid behavior

Linear conductance tensor

$$G_{jk} = e\frac{\partial I_j}{\partial \mu_k} = \frac{2e^2}{h}\left(1-\left(T/T_K\right)^{2y-2}\right)\left[\delta_{jk} - \frac{1}{M}\right]$$

asymptotic low-temperature behavior

- Non-integer scaling dimension $y = 2\left(1-\frac{1}{M}\right) > 1$

  implies non-Fermi liquid behavior
- completely isotropic multi-terminal junction

Majorana surface code

- Network of interacting Majorana fermions → realization of Majorana surface code

Terhal, Hassler & DiVincenzo, PRL 2012; Vijay, Hsieh & Fu, arXiv:1504.01724

- Surface code quantum computation
  - Encode ‚logical' qubit by entangling many physical qubits
  - Comparatively simple 2D array layouts
  - Error tolerance orders of magnitude better than in alternative approaches
  - Error detection without need for active error correction & controlled by classical software

Review: Fowler, Mariantoni, Martinis & Clarke, PRA 2012

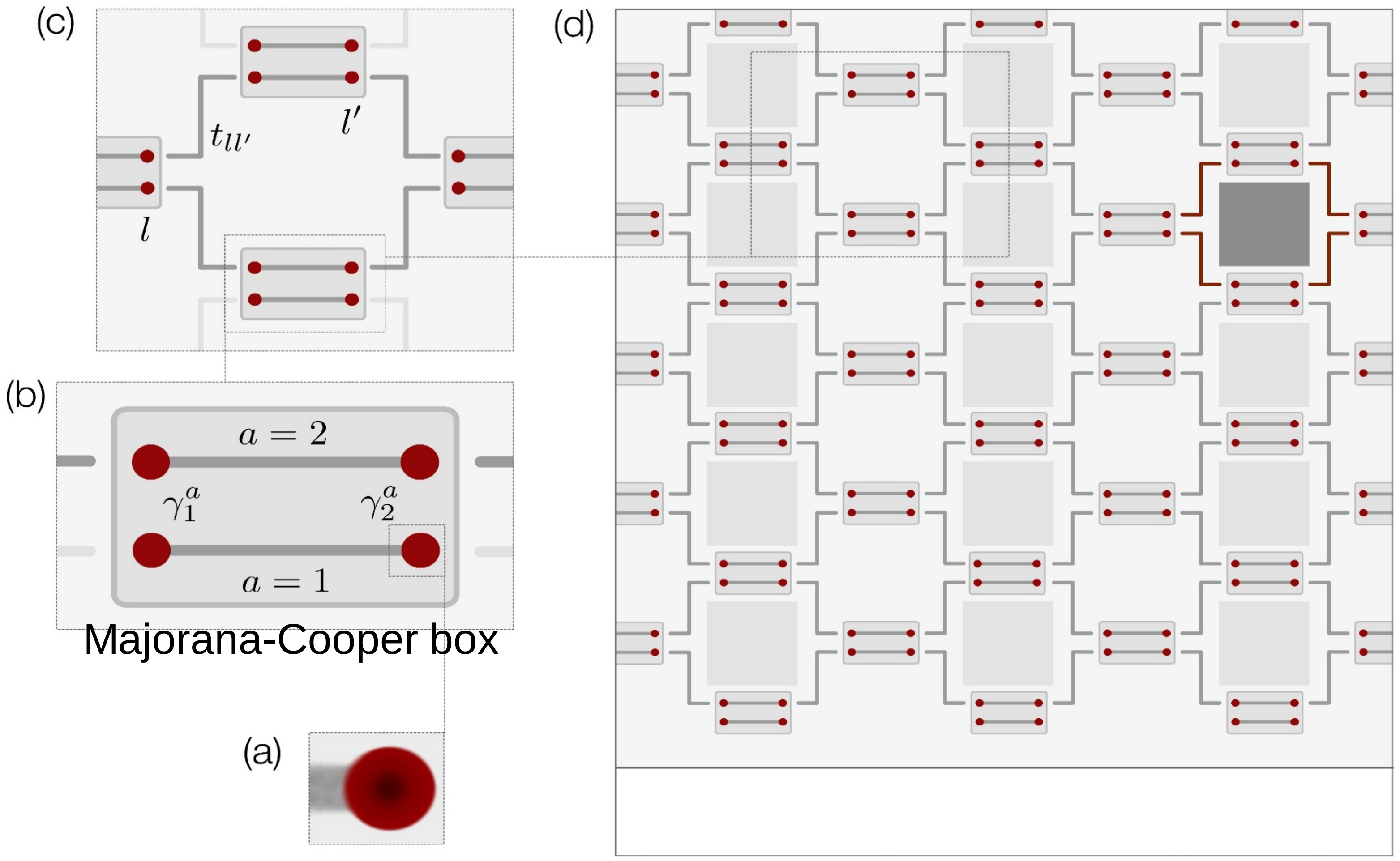
Scalability issues

- Reasonably fault-tolerant logical qubit needs $>10^3$ physical qubits $\rightarrow$ we need about 10^8 physical qubits to factorize 100-digit integer
- Maximal simplicity in implementation and access to physical qubits required
- Semiconductor Majorana layouts may offer this simplicity
  - Single-step readout without ancilla qubits possible

    Vijay, Hsieh & Fu, arXiv:1504.01724
  - Qubit readout & manipulation through tunnel probes & SETs, without radiation fields or flux interferometry

    Landau, Plugge, Sela, Altland, Albrecht & Egger, preprint

Blueprint: 2D array of boxes



2D array of Majorana-Cooper boxes

- Building block: Majorana-Cooper box with two proximitized helical nanowires
  - joined by superconductor slab: finite common charging energy
  - each box hosts M=4 Majorana zero modes
    - All wires in array parallel: homogeneous Zeeman field $\rightarrow$ simultaneous topological transition
- Now couple neighboring boxes by tunnel bridges

$$H_t = -\frac{t_{ll'}}{2}\gamma_l \gamma_{l'} e^{i(\varphi_l - \varphi_{l'})/2} + \text{h.c.}$$

Stabilizers: plaquette operators

- Low-energy excitations of array correspond to minimal loop structures
  - Minimal loop contains 8 Majorana operators
- Hermitian plaquette operator for loop no. *n*

$$O_n = \prod_{j=1}^{8} \gamma_j^{(n)}$$

- Set of mutually commuting operators
  - Plaquette eigenvalues = ±1: simultaneously measurable set of physical qubits
  - serve as stabilizers of the surface code

Plaquette Hamiltonian

- Schrieffer-Wolff transformation → low-energy theory

$$H_{code} = -\sum_n \mathrm{Re}\left(c_n\right) O_n \qquad c_n = \frac{5}{16} \frac{\prod t_{l_n l_n'}}{E_C^3}$$

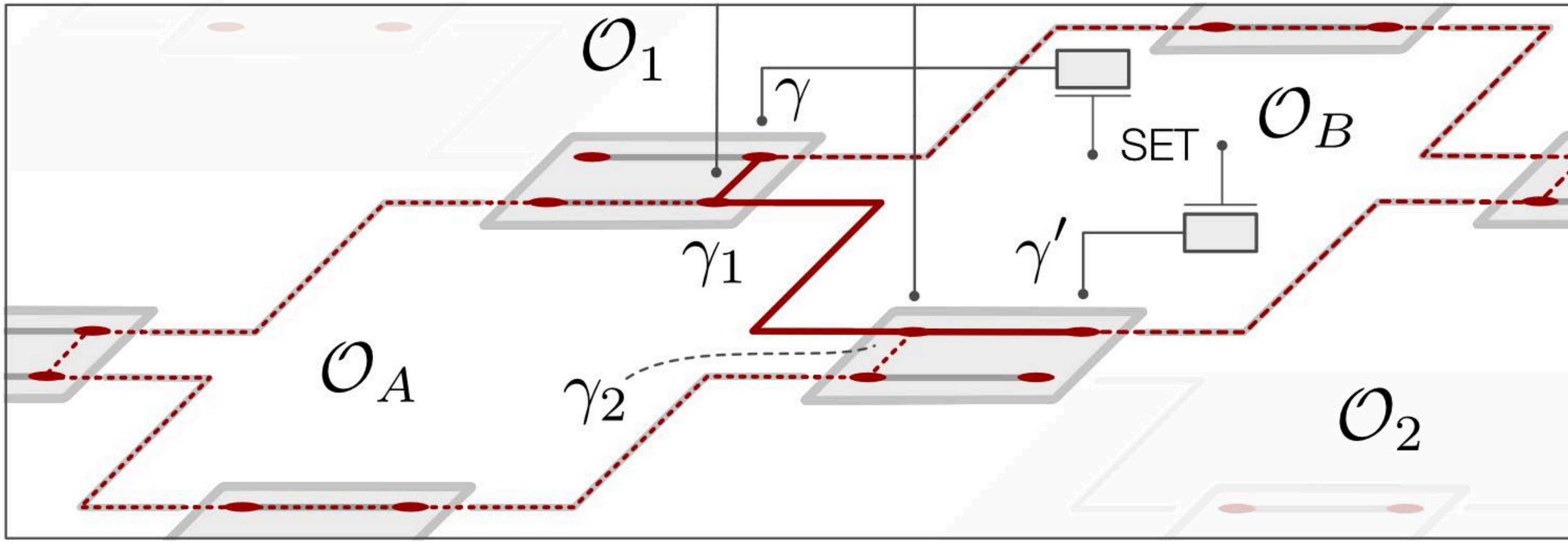
  - Amplitude c_n for n-th loop contains product of four tunnel amplitudes along loop
  - Surface code works although the Re(c_n) are generally uncorrelated random energies
- How to measure and manipulate plaquettes?
  - Essential ingredient for surface code quantum information processing

Surface code operation principles

Fowler et al., PRA 2012

- Permanent repetition of sequential stabilizer measurements
  - Project code onto eigenstate of stabilizer system
  - Occasional erroneous plaquette flips are simply recorded (& corrected by classical software), no active error correction needed
  - Punching „holes“ by ceasing measurements at plaquette(s): binary eigenstates of Wilson loops around holes serve as logical qubits
- need maximally simple readout & controlled flip of stabilizers

Attaching tunnel probes and SETs



Read out of stabilizers: tunnel conductance via attached leads, noninvasive (no plaquette flip) measurement

Controlled plaquette flip: Transfer 1 electron through code by changing gate voltages on attached pair of SETs

Tunnel conductance

Attach pair of tunnel contacts (normal leads)

$$H_{tunn} = \sum_{j=1,2} \lambda_j \psi_j^+(0)\; e^{-i\varphi_j/2} \gamma_j + \text{h.c.}$$

γ_j anticommutes with the two O_n containing γ_j

... and commutes with all other O_n

Neighboring $\gamma_{1,2}$ belong to same O_n pair

→ double flip, i.e., all plaquettes remain invariant

→ this specific conductance measurement is noninvasive

Quantitative results for conductance

Schrieffer-Wolff transformation with leads : effective coupling Hamiltonian

$$H_{eff} = \alpha \left(\xi + c_A^* O_A + c_B O_B\right) \psi_1^+(0)\, \psi_2(0) + \text{h.c.}$$

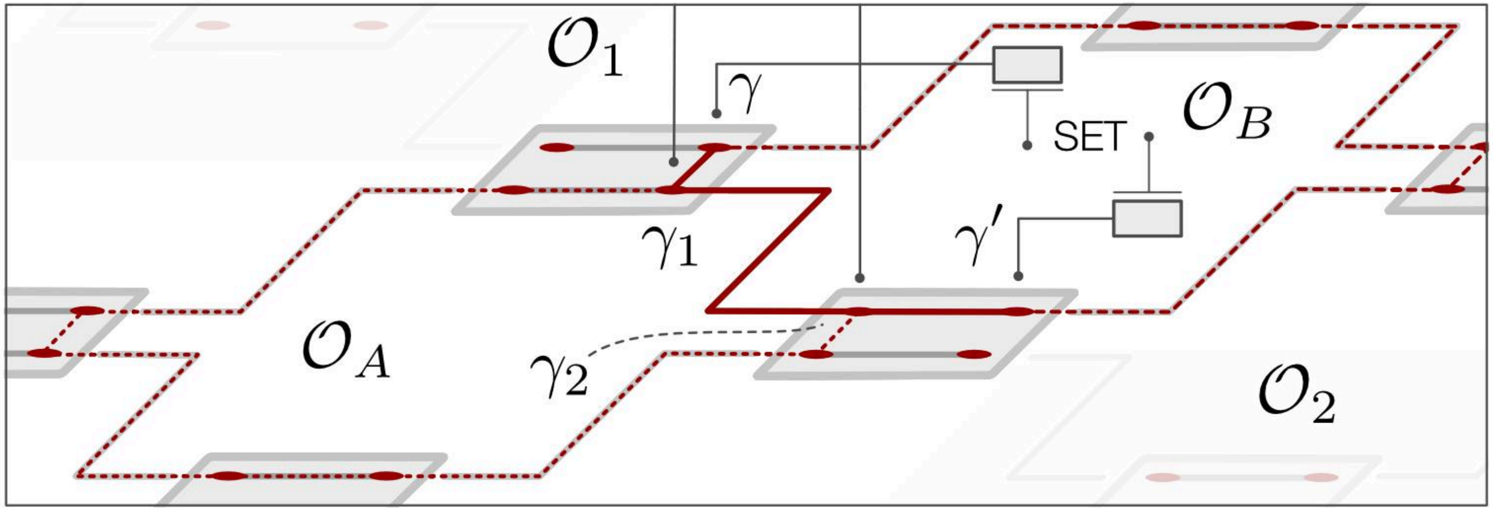
$$\alpha = -\frac{32}{5} \frac{\lambda_1 \lambda_2^*}{t_{12}^* E_C} \qquad c_n = \frac{5}{16} \frac{\prod t_{l_n l_n'}}{E_C^3}$$

- two paths around plaquettes A & B
- „direct“ amplitude ξ from 1→2 vanishes for integer n_g (but finite away from valley center)
- Tunnel conductance from lead 1 → 2 follows from perturbation theory

Tunnel conductance

Interference terms: tunnel conductance is sensitive to A and B plaquette eigenvalues

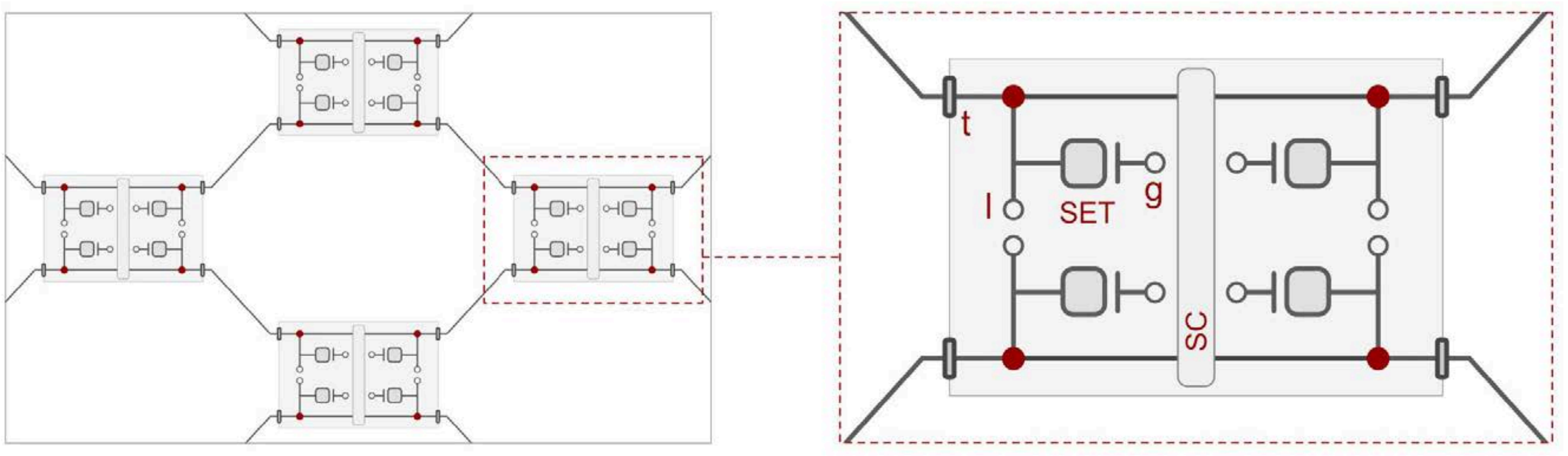
$$G_{12} \propto |\alpha|^2 \left(g_0 + g_A O_A + g_B O_B + g_{AB} O_A O_B \right)$$



Manipulation: flipping plaquettes

- Use pair of SETs to adiabatically pump single electron through array, tunneling in (out) at γ (γ')
  - Change SET configuration (1,0) $\rightarrow$ (0,1) via gate voltages *Flensberg, PRL 2011*
  - Arbitrary Majorana pair has 0, 1, or 2 plaquettes in common
  - This electron transfer flips 4, 2, or 0 plaquettes
    - No plaquette flipped: recover non-invasive case
    - Minimal excitation: 2 flipped plaquettes, cf. figure
- Activation of SET pairs at arbitrary places:
  create and move excitations arbitrarily

Towards hardware layout



Conclusions

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THANK YOU FOR YOUR ATTENTION!