Multi-terminal Coulomb-Majorana Junction

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Overview

Coulomb charging effects on quantum transport through Majorana nanowires:

- Two-terminal device: Majorana single-charge transistor with two Majorana bound states

  *Zazunov, Levy Yeyati & Egger, PRB **84**, 165440 (2011)*

  *Hützen, Zazunov, Braunecker, Levy Yeyati & Egger, PRL **109**,166403 (2012)*

- Multi-terminal device (M>2 Majoranas): approaching a non-Fermi liquid SO(M) Kondo model

  *Altland & Egger, PRL **110**, 196401 (2013)*

  Zazunov, Altland & Egger, arXiv:1307.0210

Majorana bound states

Beenakker, Ann. Rev. Con. Mat. Phys. 2013
Alicea, Rep. Prog. Phys. 2012
Leijnse & Flensberg, Semicond. Sci. Tech. 2012

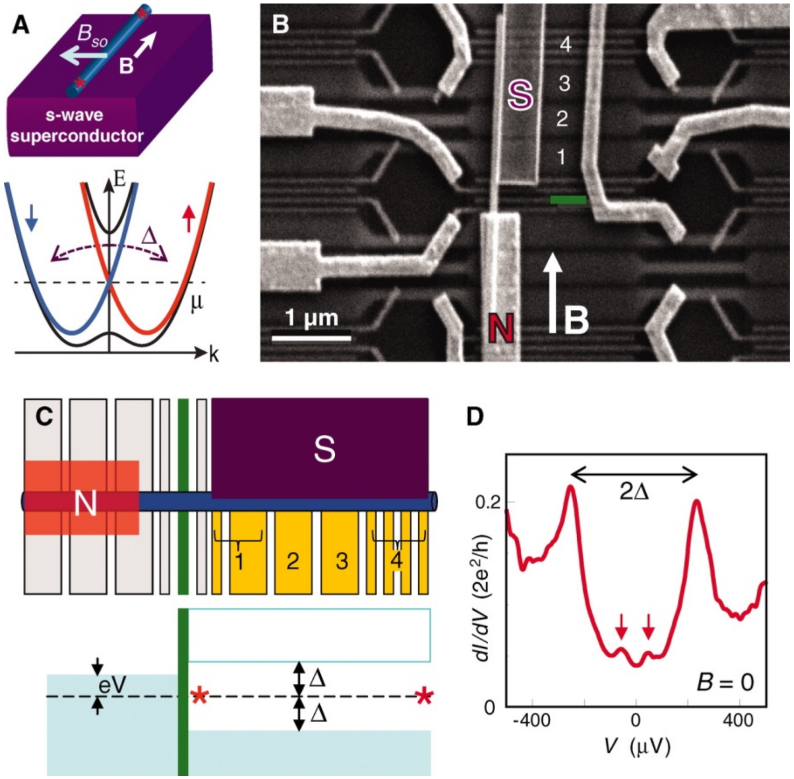
- Majorana fermions
  - Non-Abelian exchange statistics $\gamma_j = \gamma_j^+ \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$
  - Two Majoranas = nonlocal fermion $d = \gamma_1 + i\gamma_2$
  - Occupation of single Majorana ill-defined: $\gamma^+\gamma = \gamma^2 = 1$
  - Count state of Majorana pair $d^+d = 0,1$
- Realizable (for example) as end states of spinless 1D p-wave superconductor (Kitaev chain)
  - Recipe: Proximity coupling of 1D helical wire to s-wave superconductor
  - For long wires: Majorana bound states are zero energy modes

Experimental Majorana signatures

InSb nanowires expected to host Majoranas due to interplay of
- strong Rashba spin orbit field
- magnetic Zeeman field
- proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

Mourik et al., Science 2012



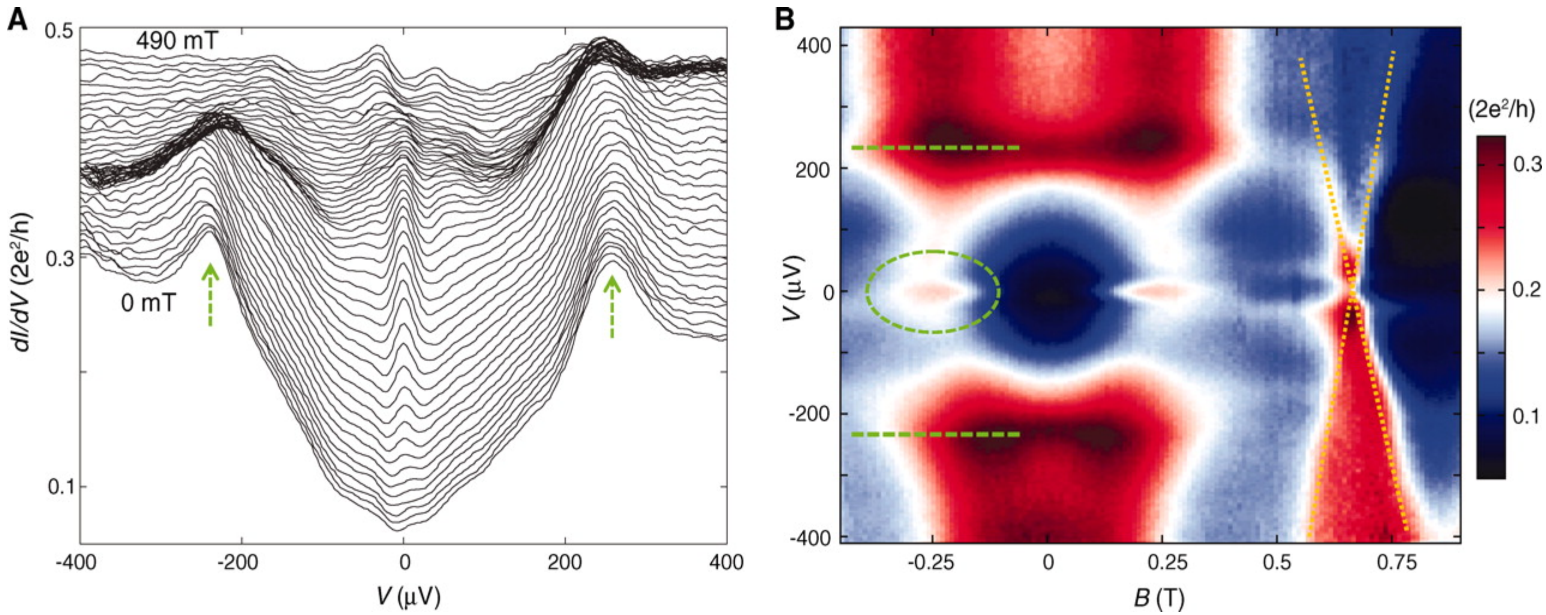
Transport signature of Majoranas: Zero-bias conductance peak due to resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009
Flensberg, PRB 2010

See also: *Rokhinson et al., Nat. Phys. 2012; Deng et al., Nano Lett. 2012; Das et al., Nat. Phys. 2012; Churchill et al., PRB 2013*

Zero-bias conductance peak

Mourik et al., Science 2012

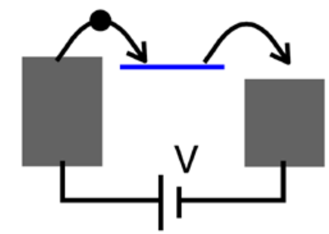


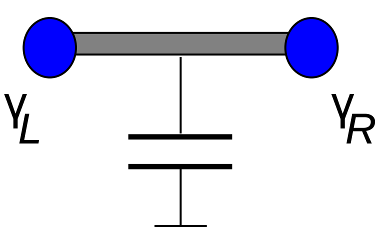
Possible explanations:

- Majorana state (most likely!)
- Disorder-induced peak — *Bagrets & Altland, PRL 2012*
- Smooth confinement — *Kells, Meidan & Brouwer, PRB 2012*
- Kondo effect — *Lee et al., PRL 2012*

Suppose that Majorana mode is realized…

- Quantum transport features beyond zero-bias anomaly peak? Coulomb interaction effects?
- Simplest case: Majorana single charge transistor
  - ‚Overhanging' helical wire parts serve as normal-conducting leads
  - Nanowire part coupled to superconductor hosts pair of Majorana bound states
  - Include charging energy of this ‚dot'

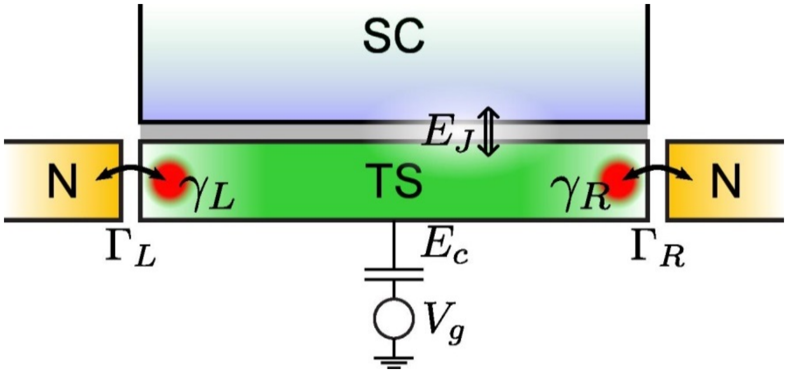




Majorana single charge transistor

Hützen et al., PRL 2012

- Floating superconducting ‚dot‘ contains two Majorana bound states tunnel-coupled to normal-conducting leads
- Charging energy finite



- Consider universal regime:
  - Long superconducting wire:
    Direct tunnel coupling between left and right Majorana modes is assumed negligible
  - No quasi-particle excitations:
    Proximity-induced gap is largest energy scale of interest

Hamiltonian: charging term

- Majorana pair: nonlocal fermion $d = \gamma_L + i\gamma_R$
- Condensate gives another zero mode
  - Cooper pair number N_c, conjugate phase ϕ
- Dot Hamiltonian (gate parameter n_g)

$$H_c = E_c\left(2N_c + d^+d - n_g\right)^2$$

Majorana fermions couple to Cooper pairs through the charging energy

Tunneling

- Normal-conducting leads: noninteracting fermions (effectively spinless helical wire)
  - Applied bias voltage V = chemical potential difference
- Tunneling of electrons from lead to dot:
  - Project electron operator in superconducting wire part to Majorana sector
  - Spin structure of Majorana state encoded in tunneling matrix elements

Flensberg, PRB 2010

Tunneling Hamiltonian

Source (drain) couples to left (right) Majorana only:

$$H_t = \sum_{j=L,R} t_j c_j^+ \eta_j + h.c. \qquad \eta_j = \left(d \pm e^{-i\phi} d^+\right)/2$$

- respects current conservation
- Hybridizations: $\Gamma_{L/R} \sim \rho_0 \left| t_{L/R} \right|^2$

Normal tunneling $\sim c^+ d, d^+ c$

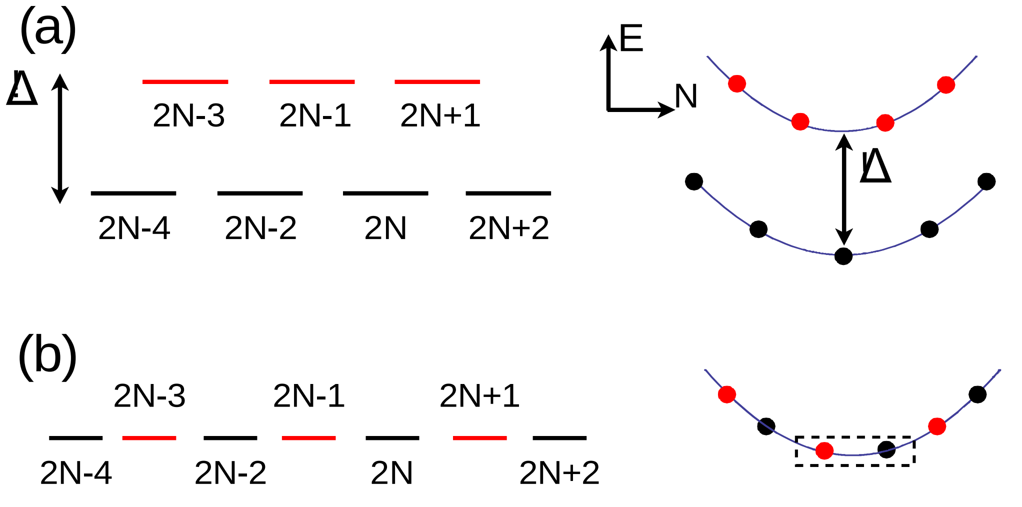
- Either destroy or create nonlocal d fermion
- Condensate not involved

Anomalous tunneling $\sim c^+ e^{-i\phi} d^+, d e^{i\phi} c$

- Create (destroy) both lead and d fermion & split (add) a Cooper pair

Absence of even-odd effect

- Without Majorana states: Even-odd effect
- With Majoranas: no even-odd effect!
  - Tuning wire parameters into the topological phase removes even-odd effect



picture from: Fu, PRL 2010

Majorana Meir-Wingreen formula

- Exact expression for interacting Majorana dot

$$I_{j=L,R} = \frac{e\Gamma_j}{h}\int d\varepsilon\ F\left(\varepsilon-\mu_j\right)\operatorname{Im} G^{ret}_{\eta_j}\left(\varepsilon\right)$$

  - Lead Fermi distribution encoded in $F(\varepsilon) = \tanh\left(\varepsilon/2T\right)$
  - Proof uses $\eta_j^+\eta_j = 1$
- Differential conductance: $G = dI/dV$

$$I = \left(I_L - I_R\right)/2$$

Here: symmetric case $\Gamma_L = \Gamma_R = \Gamma/2$

Noninteracting case: Resonant Andreev reflection

Bolech & Demler, PRL 2007
Law, Lee & Ng, PRL 2009

- E_c=0 Majorana spectral function

$$-\operatorname{Im} G^{ret}_{\gamma_j}(\varepsilon) = \frac{\Gamma_j}{\varepsilon^2 + \Gamma_j^2}$$

- T=0 differential conductance:

$$G(V) = \frac{2e^2}{h} \frac{1}{1 + (eV/\Gamma)^2}$$

- Currents I_L and I_R fluctuate independently, superconductor is effectively grounded
- Perfect Andreev reflection via Majorana state
  - Zero-energy Majorana bound state leaks into lead

Strong blockade: Electron teleportation

Fu, PRL 2010

- Peak conductance for half-integer n_g
- Strong charging energy then allows only two degenerate charge configurations
- Model maps to spinless resonant tunneling model
- Linear conductance (T=0): $G = e^2 / h$
- Interpretation: Electron teleportation due to nonlocality of d fermion

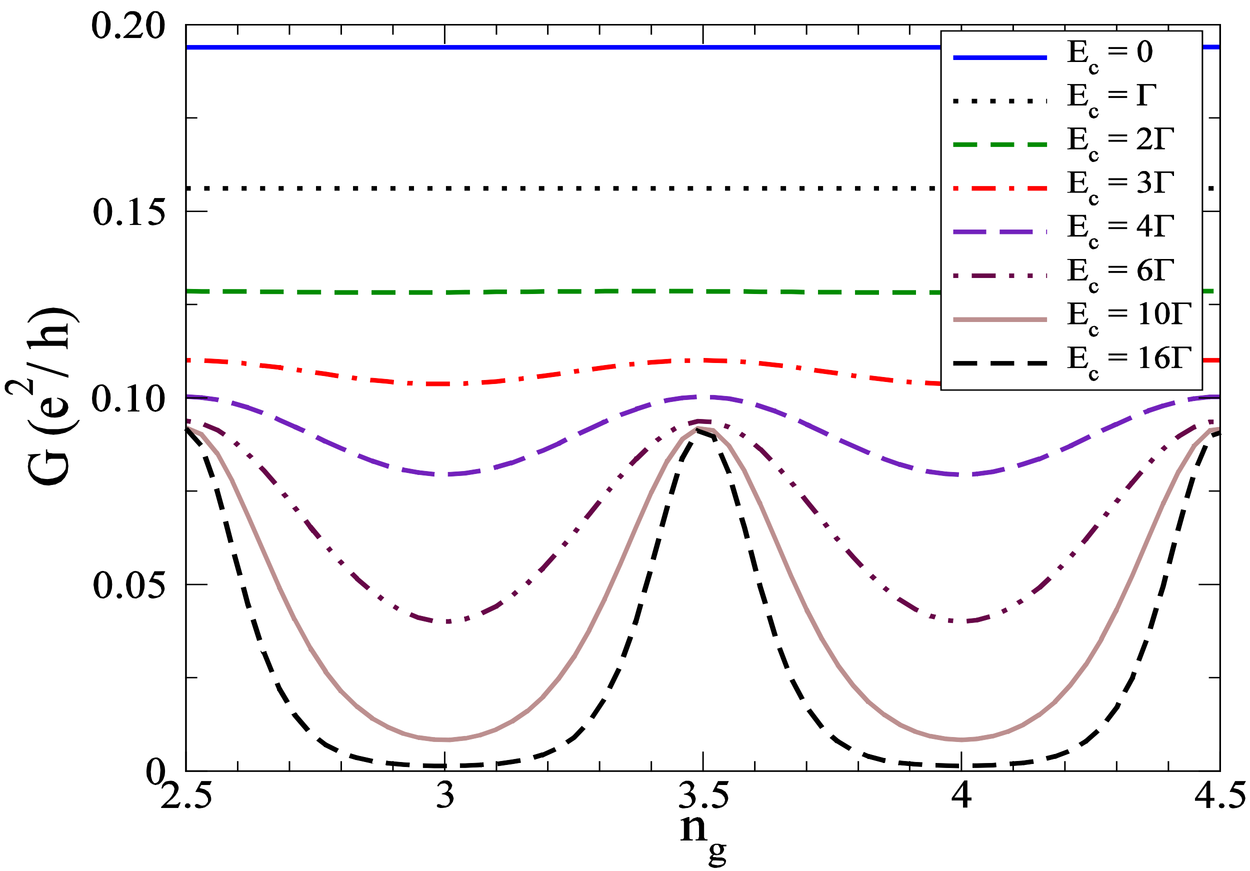
Crossover from resonant Andreev reflection to electron teleportation

- Keldysh approach yields full action in phase representation *Zazunov, Levy Yeyati & Egger, PRB 2011*
  - Practically useful in weak Coulomb blockade regime: interaction corrections to conductance
- Full crossover from three other methods:

Hützen et al., PRL 2012

  - Master equation for $T>\Gamma$: include sequential and all cotunneling processes (incl. local and crossed Andreev reflection)
  - Equation of motion approach for peak conductance
  - Zero bandwidth model for leads: exact solution

Coulomb oscillations

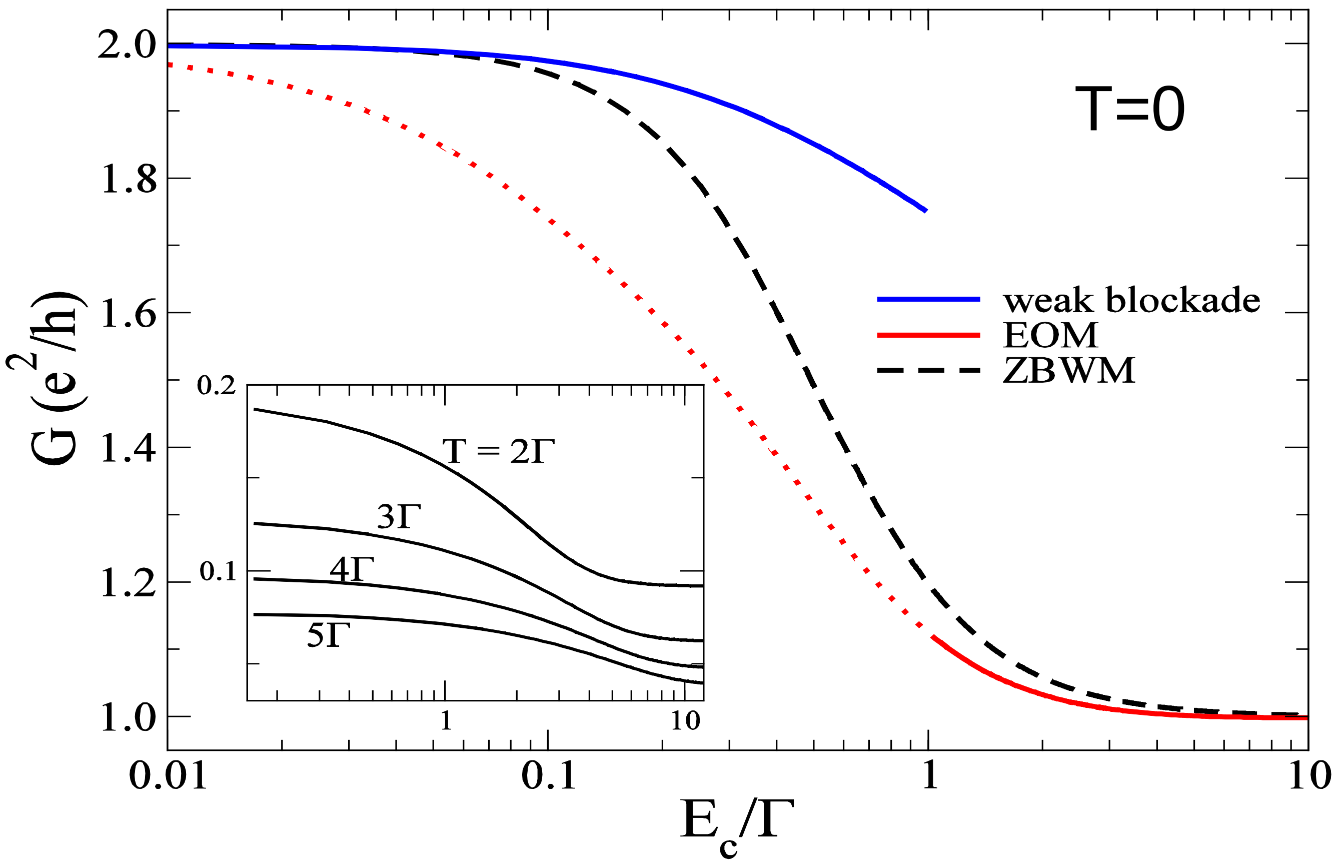


Master equation

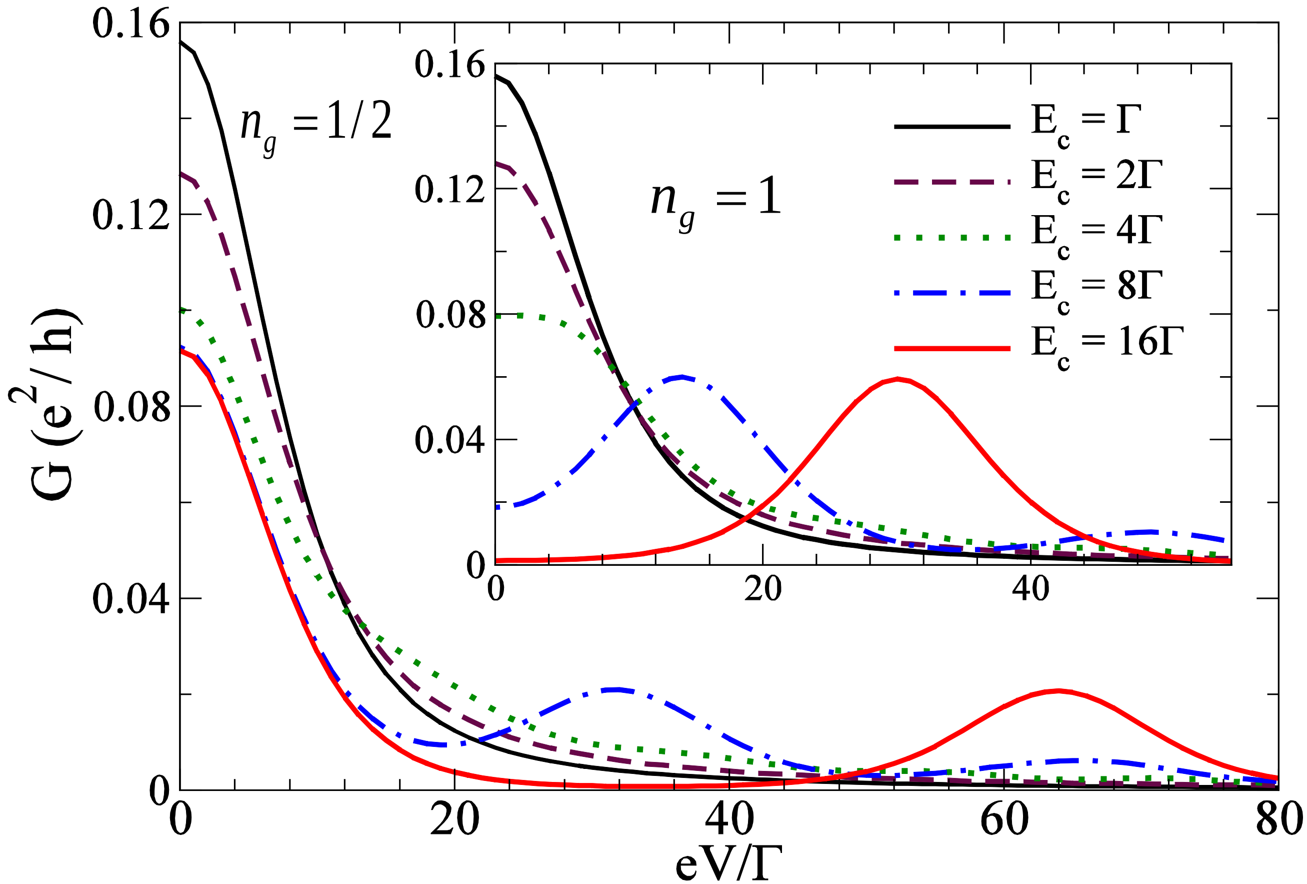
$T = 2\Gamma$

Valley conductance dominated by elastic cotunneling

Peak conductance: from resonant Andreev reflection to teleportation



Finite bias sidepeaks



Master equation

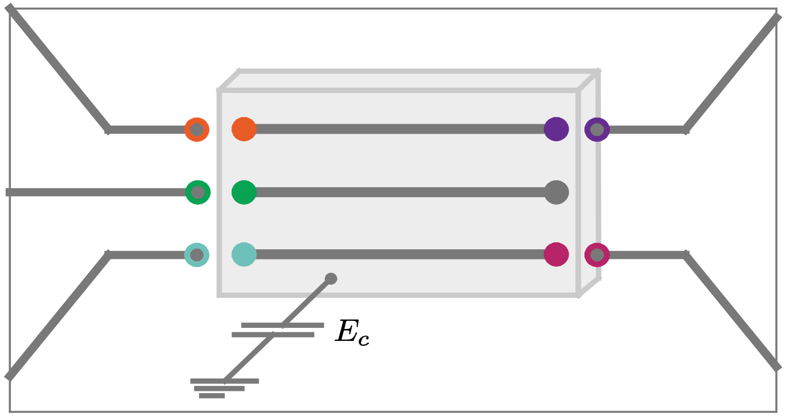
$T = 2\Gamma$

Finite bias sidepeaks

- On resonance: sidepeaks at $eV = 4nE_c$
  - $\mu_{L,R}$ resonant with two (almost) degenerate higher order charge states: additional sequential tunneling contributions
  - Requires change of Cooper pair number - only possible through anomalous tunneling:

    without Majoranas no side-peaks
- Similar sidepeaks away from resonance

Multiterminal Coulomb-Majorana junction



Zazunov, Altland & Egger, arXiv:1307.0210
Altland & Egger, PRL 2013
Beri & Cooper, PRL 2012; Beri, PRL 2013
Tsvelik, PRL 2013

- Now N>1 helical wires: M Majoranas modes, tunnel-coupled to helical Luttinger liquid wires, g≤1
- Bosonization of leads: Klein-Majorana fusion
  - Klein factors → additional Majorana fermion for each lead
  - Combine Klein-Majorana and ‚true‘ Majorana at each contact to build fermion f_j
  - All occupation numbers $f_j^+ f_j$ are conserved and can be gauged away

Charging effects: dipole confinement

- High energy scales $> E_c$: charging effects irrelevant
  - Electron tunneling amplitudes from lead j to dot renormalize independently upwards
    $$t_j(E) \sim E^{-1+\frac{1}{2g}}$$
  - RG flow towards resonant Andreev reflection fixed point
- For $E < E_c$: charging induces ‚confinement'
  - In- and out-tunneling events are bound to ‚dipoles' with coupling $\lambda_{j\neq k}$: entanglement of different leads
  - Dipole coupling describes amplitude for ‚teleportation' from lead j to lead k
  - ‚Bare' value
    $$\lambda_{jk}^{(1)} = \frac{t_j(E_c)\, t_k(E_c)}{E_c} \sim E_c^{-3+\frac{1}{g}}$$
    large for small E_c

RG equations in dipole phase

- Energy scales below E_c: effective phase action

$$S = S_{Lutt}[\Phi] - \sum_{j\neq k} \lambda_{jk} \int d\tau \cos(\Phi_j - \Phi_k)$$

- One-loop RG equations

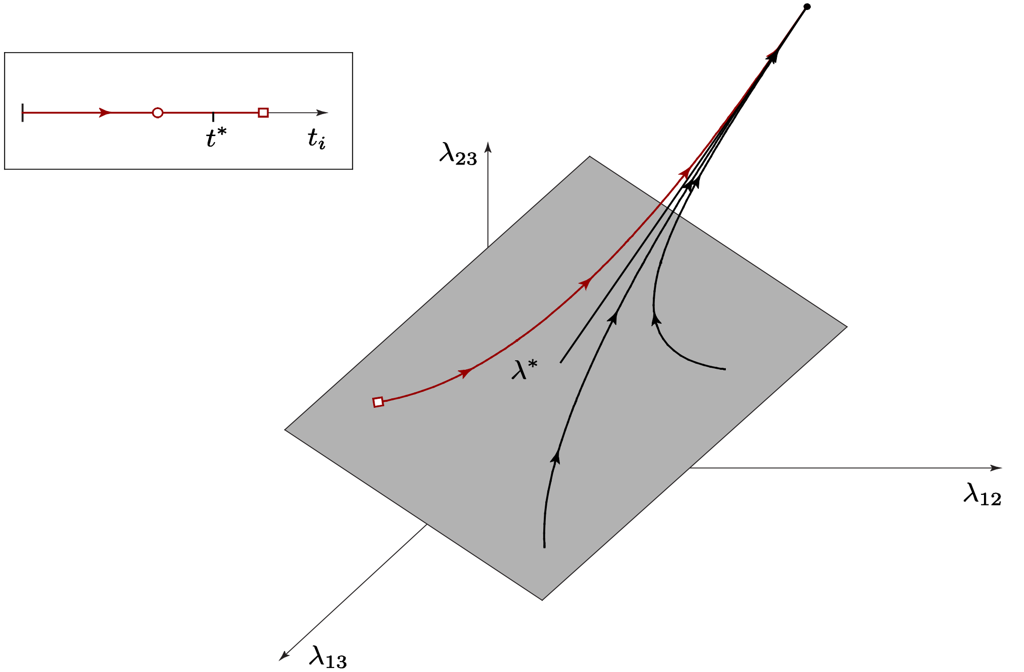
$$\frac{d\lambda_{jk}}{dl} = -(g^{-1} - 1)\lambda_{jk} + E_c^{-1} \sum_{m\neq(j,k)}^{M} \lambda_{jm}\lambda_{mk}$$

  - suppression by Luttinger tunneling density of states
  - enhancement by dipole fusion events

- RG-unstable intermediate fixed point with isotropic couplings (for M>2 leads)

$$\lambda_{j\neq k} = \lambda^* = \frac{g^{-1} - 1}{M - 2} E_c$$

RG flow



- RG flow towards strong coupling for $\left\langle \lambda^{(1)} \right\rangle > \lambda^*$ for moderate charging energy
- Flow towards isotropic couplings, anisotropies are RG-irrelevant
- Perturbative RG fails below Kondo temperature $T_K \approx E_c e^{-\lambda^* / \left\langle \lambda^{(1)} \right\rangle}$

Topological Kondo effect

Beri & Cooper, PRL 2012

- Refermionization of g=1 dipole action:

$$H = -i\int_{-\infty}^{\infty} dx \sum_{j=1}^{M} \Psi_j^{+} \partial_x \Psi_j - \lambda \sum_{j\neq k} \Psi_j^{+}(0) A_{jk} \Psi_k(0)$$

- Majorana bilinears $A_{jk} \equiv \gamma_j \gamma_k$ define so(M) algebra: nonlocal ‚impurity spin'
- Majorana basis $\Psi(x) = \mu + i\xi$ for leads: two-channel SO(M) Kondo model

$$H = -i\int dx \mu^T \partial_x \mu - \lambda \mu^T(0) \hat{A} \mu(0) + [\mu \leftrightarrow \xi]$$

Transport properties near unitary limit

- Temperature & voltages < T_K:
  - Dual instanton version of dipole action: study behavior near strong coupling limit
  - Nonequilibrium Keldysh formulation
- Linear conductance tensor

$$G_{jk} = -e\frac{\partial I_j}{\partial \mu_k} = \frac{2ge^2}{h}\left(1-\left(T/T_K\right)^{2\Delta-2}\right)\left[\delta_{jk} - \frac{1}{M}\right]$$

  - Non-integer scaling dimension $\Delta = 2g\left(1-\frac{1}{M}\right) > 1$ implies non-Fermi liquid behavior even for g=1
  - completely isotropic multiterminal junction

Fano factor

Zazunov et al., arXiv:1307.0210

- Backscattering correction to current near unitary limit for $\sum_j \mu_j = 0$

$$\delta I_j = -\frac{e}{\hbar}\sum_k \left|\frac{\mu_k}{T_K}\right|^{2\Delta-2}\left(\delta_{jk} - \frac{1}{M}\right)\mu_k$$

- Shot noise: $S_{jk}(\omega \to 0) = \int dt\, e^{i\omega t}\left(\langle I_j(t) I_k(0)\rangle - \langle I_j\rangle\langle I_k\rangle\right)$

$$S_{jk} = -\frac{2ge^2}{\hbar}\sum_l \left(\delta_{jl} - \frac{1}{M}\right)\left(\delta_{kl} - \frac{1}{M}\right)\left|\frac{\mu_l}{T_K}\right|^{2\Delta-2}|\mu_l|$$

- universal Fano factor but different value than for SU(N) Kondo effect

Sela et al. PRL 2006; Mora et al., PRB 2009

Conclusions

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*Altland & Egger, PRL **110**, 196401 (2013)*
Zazunov, Altland & Egger, arXiv:1307.0210