Tunneling through a Luttinger dot

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Overview

- Intro: Luttinger liquid behavior in SWNTs
- Tunneling through a double barrier (Luttinger liquid dot)
- Correlated sequential tunneling: Master equation approach
- Real-time Monte Carlo simulations
- Conclusions
Ballistic SWNTs as 1D quantum wires

- Transverse momentum quantization: only one relevant transverse mode, all others are far away from Fermi surface
- 1D quantum wire with two spin-degenerate transport channels (bands)
- Linear dispersion relation for metallic SWNTs
- Effect of electron-electron interactions on transport properties?
Field theory: clean interacting SWNTs

- Keep only two bands at Fermi energy
- Low-energy expansion of electron operator in terms of Bloch states introduces 1D fermions
- 1D fermion operators: Bosonization applies, and allows to include Coulomb interactions nonperturbatively
- Four channels: $c^+, c^-, s^+, s^-$

Egger & Gogolin, PRL 1997, EPJB 1998
Kane, Balents & Fisher, PRL 1997
Effective 1D interaction processes

Momentum conservation allows only two processes away from half-filling

- **Forward scattering**: „Slow“ density modes, probes long-range part of interaction
- **Backscattering**: „Fast“ density modes, probes short-range properties of interaction
- Backscattering couplings \( f,b \) scale as \( 1/R \), sizeable only for ultrathin tubes
- SWNT then described by **Luttinger liquid model**, with exotic properties (fractionalization, spin-charge separation, no Landau quasiparticles)
Luttinger parameters for SWNTs

- Interaction strength encoded in dimensionless Luttinger parameters
- Bosonization gives $g_{a\neq c^+} \approx 1$
- Logarithmic divergence for unscreened interaction, cut off by tube length
  \[ g_{c^+} = \left[ 1 + \frac{8e^2}{\pi^2 \hbar v_F} \ln\left( \frac{L}{2\pi R} \right) \right]^{-1/2} = \frac{1}{\sqrt{1 + 2E_c / \Delta}} \approx 0.2 \]
- Pronounced non-Fermi liquid correlations!
Tunneling DoS for nanotube

- Power-law suppression of tunneling DoS reflects orthogonality catastrophe: Electron has to decompose into true quasiparticles

- Explicit calculation gives

\[ \nu(x, E) = \text{Re} \int_{0}^{\infty} dt e^{iEt} \langle \Psi(x, t) \Psi^+(x, 0) \rangle \propto E^\eta \]

- Geometry dependence:

\[ \eta_{bulk} = (g + 1 / (g - 2)) / 4 \]

\[ \eta_{end} = (1 / (g - 1)) / 2 > 2 \eta_{bulk} \]
Mounting evidence for Luttinger liquid in single-wall nanotubes

- Tunneling density of states (many groups)
- **Double barrier tunneling** \( \text{Postma et al., Science 2001} \)
- Transport in crossed geometry (no tunneling) \( \text{Gao, Komnik, Egger, Glattli & Bachtold, PRL 2004} \)
- Photoemission spectra (spectral function) \( \text{Ishii, Kataura et al., Nature 2003} \)
- STM probes of density pattern \( \text{Lee et al. PRL 2004} \)
- Spin-charge separation & fractionalization so far not observed in nanotubes!
Tunneling through a double barrier: Experimental data

Postma et al., Science 2001

Power law scaling of the peak conductance
Signature of Luttinger liquid?

- Power law in temperature-dependence of the peak conductance smells like Luttinger liquid
  - Usual (Fermi liquid) dots: \( G_{\text{max}} \propto T^{-1} \)
- Effective single-channel model (charge sector)
  \[
  \frac{1}{g} = \frac{1}{4} \left( 3 + \frac{1}{g_{c+}} \right) \approx 0.55
  \]
- Sequential tunneling regime (high temperature, weak transmission):  
  Master (rate) equation approach
- Focus on peak linear conductance only
Luttinger model with double barrier

- Bosonized Hamiltonian

\[
H = \frac{\nu}{2} \int dx \left[ \Pi^2 + g^{-2} (\partial_x \phi)^2 \right] + V_0 \sum_{x_{\text{imp}} = \pm d/2} ^\pi \frac{\cos(\sqrt{4\pi} \phi(x_{\text{imp}}))}{2}\] + H_{\text{ext}}

- Hybridization: \[ \Gamma = 2\pi \nu \Delta^2 \]

for hopping matrix element \[ \Delta \propto V_0^{-1/g} \]

Away from barriers: Gaussian model
Dual tight-binding representation

- Integrate out all Luttinger fields away from barriers, dissipative bath for remaining degrees of freedom $N,n$
  - $-\epsilon N$: charge difference between left and right lead
  - $-\epsilon n$: charge on the island (dot)
- Maps double-barrier Luttinger problem to coupled **Quantum Brownian motion** of $N,n$ in 2D periodic potential
  - Coulomb blockade peak: Only $n=0,1$ possible
Master equation: Rate contributions

Expansion in lead-to-dot hopping $\Delta$, visualized in reduced density matrix

(a) UST

(b) COT

Lowest-order sequential tunneling
(Golden Rule diagram)

Furusaki, PRB 1998

Cotunneling, only important away from resonance
Sequential tunneling regime

- Golden rule rate scales as: \[ \Gamma_{UST} \propto T^{-1+1/g} \]
- Implies \( T \) dependence of peak conductance:
  \[ G_{\text{max}, UST} \propto T^{-2+1/g} \]
- Differs from observed one, which is better described by the power law
  \[ G_{\text{max,exp}} \propto T^{-3+2/g} \]
  - Different sign in exponent!
- Has been ascribed to **Correlated Sequential Tunneling (CST)**

*Grifoni et al., Science 2001, PRL 2001*
A recent debate...

- CST theory of Grifoni et al. based on uncontrolled approximations
- No indication for CST power law scaling in expansions around noninteracting limit
  - Nazarov & Glazman, PRL 2003,
  - Gornyi et al., PRB 2003,
  - Meden et al., PRB 2005

What is going on?
- Master equation approach: systematic evaluation of higher order rates
- Numerically exact dynamical QMC simulations
Fourth-order rate contributions

Thorwart et al. PRB 2005

(a) indirect: 
k=7

(b) direct: CST1

Renormalization of dot lifetime

Hop from left to right without cutting the diagram on the dot:
Correlated Sequential Tunneling (CST)
Wigner-Weisskopf regularization

- CST rates *per se* divergent need regularization
- Such processes important in bridged electron transfer theory *Hu & Mukamel, JCP 1989*
- Systematic self-consistent scheme:
  - First assume finite lifetime on dot to regularize diagrams
  - Then compute lifetime self-consistently using all (up to 4th-order) rates
Detailed calculation shows:

- CST processes unimportant for high barriers
- CST processes only matter for strong interactions
- Crossover from usual sequential tunneling (UST) at high T to CST at low T
Peak conductance from Master Equation

- Crossover from UST to CST for both interaction strengths
- Temperature well below level spacing $\epsilon$
- Incoherent regime, no resonant tunneling
- No true power law scaling
- No CST for high barriers (small $\Delta$)

$$G_{\text{max}} \approx \frac{e^2}{h} \frac{\Gamma_{UST}^2}{T} \propto T^{-3+2/g}$$
Crossover temperature separating UST and CST regimes

- CST only important for strong e-e interactions
- No accessible $T$ window for weak interactions
- At very low $T$: coherent resonant tunneling

CST regime possible, but only in narrow parameter region
Real-time QMC approach

- Alternative, numerically exact approach, applicable also out of equilibrium
- Does not rely on Master equation
- Map coupled Quantum Brownian motion problem to Coulomb gas representation
- Main obstacle: Sign problem, yet asymptotic low-temperature regime can be reached

Hügle & Egger, EPL 2004
Check QMC against exact $g=1$ result

QMC reliable and accurate
Peak height from QMC

$G_p / G_0$

$\Delta = 3.3 \varepsilon$

$g = 0.6$

Sequential tunneling, CST exponent!

Coherent resonant tunneling

CST effects seen in simulation…
Strong transmission behavior

- For $k_B T / D > 0.01$ :
  - $g=1$ lineshape but with renormalized width
  
  $$w = w_g(T) \propto T^{\alpha_g}$$

- Fabry-Perot regime, broad resonance

- At lower $T$: Coherent resonant tunneling

![Graph showing $\alpha_{0.3} = 0.84$ and $\alpha_{0.6} = 0.72$ with $V_0 / D = 0.05$.](image)
Coherent resonant tunneling

Low T, arbitrary transmission:
Universal scaling

*Kane & Fisher, PRB 1992*

\[
\frac{G}{G_0} = f_g(X)
\]

\[
X = T^{g-1}\left|N_0 - 1/2\right|
\]

\[
f(X \to 0) = 1 - X^2
\]

\[
f(X \to \infty) \propto X^{-2/g}
\]
Conclusions

- Pronounced effects of electron-electron interactions in tunneling through a double barrier
- CST processes important in a narrow parameter regime, but no true CST power law scaling:
  - Intermediate barrier transparency
  - Strong interactions & low T
- Results of rate equation agree with dynamical QMC
- Estimates of parameters for Delft experiment indicate relevant regime for CST