Tunneling through a Luttinger dot

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Overview

- Intro: Luttinger liquid behavior in SWNTs
- Tunneling through a double barrier (Luttinger liquid dot)
- Correlated sequential tunneling: Master equation approach
- Real-time Monte Carlo simulations
- Conclusions

Ballistic SWNTs as 1D quantum wires

- Transverse momentum quantization:
  only one relevant transverse mode, all others are far away from Fermi surface
- 1D quantum wire with two spin-degenerate transport channels (bands)
- Linear dispersion relation for metallic SWNTs
- Effect of electron-electron interactions on transport properties?

Field theory: clean interacting SWNTs

Egger & Gogolin, PRL 1997, EPJB 1998
Kane, Balents & Fisher, PRL 1997

- Keep only two bands at Fermi energy
- Low-energy expansion of electron operator in terms of Bloch states introduces 1D fermions
- 1D fermion operators: Bosonization applies, and allows to include Coulomb interactions nonperturbatively
- Four channels: *c+,c-,s+,s-*

Effective 1D interaction processes

Momentum conservation allows only two processes away from half-filling

- Forward scattering: „Slow“ density modes, probes long-range part of interaction
- Backscattering: „Fast“ density modes, probes short-range properties of interaction
- Backscattering couplings *f,b* scale as *1/R*, sizeable only for ultrathin tubes
- SWNT then described by Luttinger liquid model, with exotic properties (fractionalization, spin-charge separation, no Landau quasiparticles)

Luttinger parameters for SWNTs

- Interaction strength encoded in dimensionless Luttinger parameters
- Bosonization gives $g_{a\neq c+} \cong 1$
- Logarithmic divergence for unscreened interaction, cut off by tube length

$$g_{c+} = \left[1 + \frac{8e^2}{\pi\kappa\hbar v_F} \ln\left(L/2\pi R\right)\right]^{-1/2} = \frac{1}{\sqrt{1+2E_c/\Delta}} \approx 0.2$$

- Pronounced non-Fermi liquid correlations!

Tunneling DoS for nanotube

- Power-law suppression of tunneling DoS reflects orthogonality catastrophe: Electron has to decompose into true quasiparticles
- Explicit calculation gives

$$\nu(x,E) = \mathrm{Re}\int_0^\infty dt e^{iEt}\left\langle \Psi(x,t)\Psi^+(x,0)\right\rangle \propto E^\eta$$

- Geometry dependence:

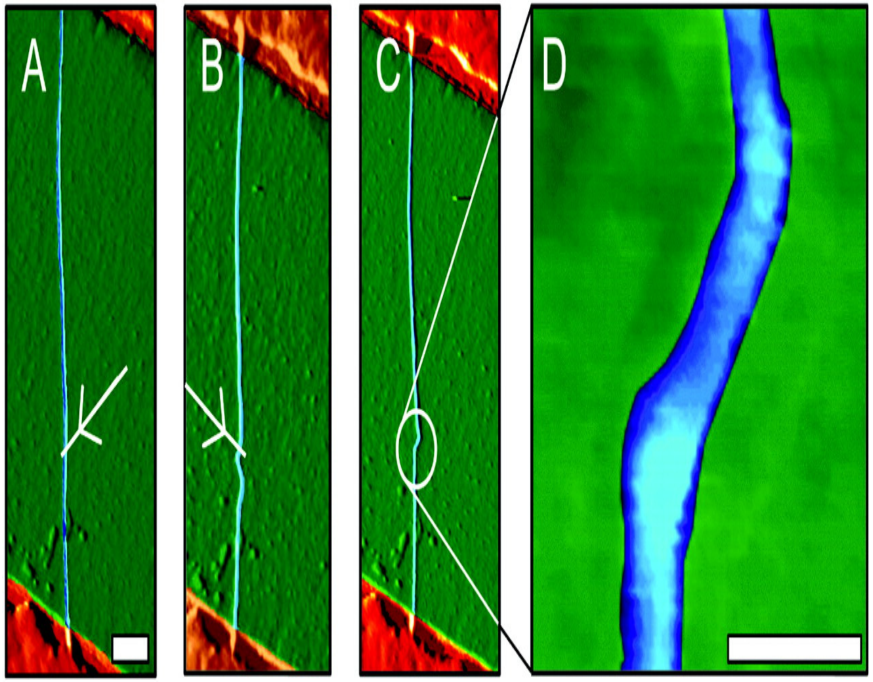
$$\eta_{bulk} = \left(g + 1/g - 2\right)/4$$

$$\eta_{end} = (1/g - 1)/2 > 2\eta_{bulk}$$

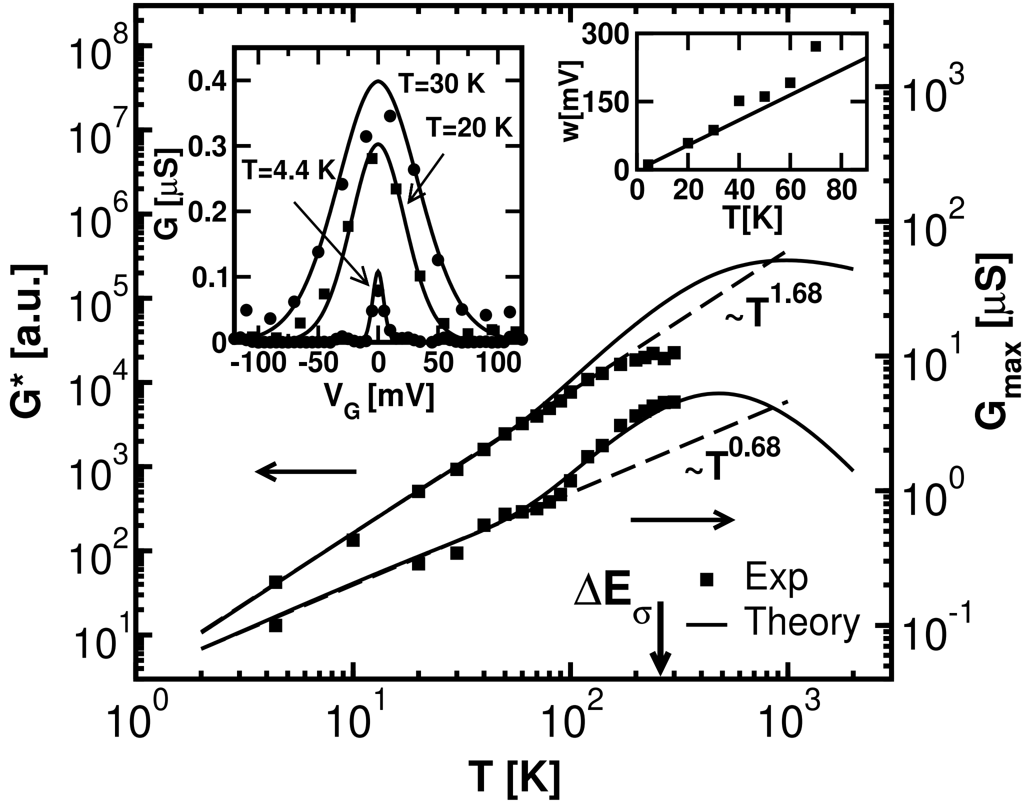
Mounting evidence for Luttinger liquid in single-wall nanotubes

- Tunneling density of states (many groups)
- Double barrier tunneling *Postma et al., Science 2001*
- Transport in crossed geometry (no tunneling)
  Gao, Komnik, Egger, Glattli & Bachtold, PRL 2004
- Photoemission spectra (spectral function)
  Ishii, Kataura et al., Nature 2003
- STM probes of density pattern *Lee et al. PRL 2004*
- Spin-charge separation & fractionalization so far not observed in nanotubes!

Tunneling through a double barrier: Experimental data



Postma et al., Science 2001



Power law scaling of the peak conductance

Signature of Luttinger liquid?

- Power law in temperature-dependence of the peak conductance smells like Luttinger liquid
  - Usual (Fermi liquid) dots: $G_{\max} \propto T^{-1}$
- Effective single-channel model (charge sector)

$$\frac{1}{g} = \frac{1}{4}\left(3 + \frac{1}{g_{c+}}\right) \approx 0.55$$

- Sequential tunneling regime (high temperature, weak transmission):

  Master (rate) equation approach
- Focus on peak linear conductance only

Luttinger model with double barrier

- Bosonized Hamiltonian

$$H = \frac{v}{2}\int dx\left[\Pi^2 + g^{-2}(\partial_x \varphi)^2\right] + V_0 \sum_{x_{imp}=\pm d/2} \cos\left(\sqrt{4\pi}\,\varphi(x_{imp})\right) + H_{ext}$$

- Hybridization: $\Gamma = 2\pi v \Delta^2$

for hopping matrix element $\Delta \propto V_0^{-1/g}$

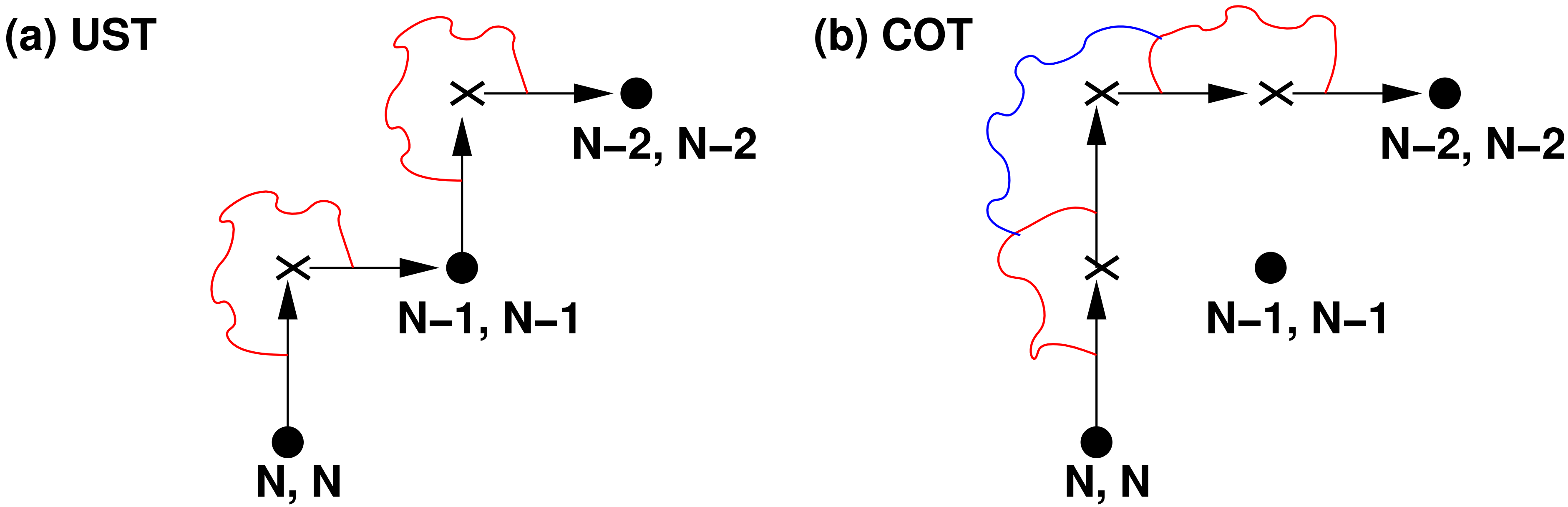
Away from barriers: Gaussian model

Dual tight-binding representation

- Integrate out all Luttinger fields away from barriers → dissipative bath for remaining degrees of freedom *N,n*
  - *-eN*: charge difference between left and right lead
  - *-en*: charge on the island (dot)
- Maps double-barrier Luttinger problem to coupled Quantum Brownian motion of *N,n* in 2D periodic potential
  - Coulomb blockade peak: Only *n=0,1* possible

Master equation: Rate contributions

Expansion in lead-to-dot hopping Δ, visualized in reduced density matrix



Lowest-order sequential tunneling
(Golden Rule diagram)

Furusaki, PRB 1998

Cotunneling, only important
away from resonance

Sequential tunneling regime

- Golden rule rate scales as $\Gamma_{UST} \propto T^{-1+1/g}$
- Implies *T* dependence of peak conductance:

$$G_{\max,\, UST} \propto T^{-2+1/g}$$

- Differs from observed one, which is better described by the power law $G_{\max,\exp} \propto T^{-3+2/g}$
  - Different sign in exponent!
- Has been ascribed to Correlated Sequential Tunneling (CST) *Grifoni et al., Science 2001, PRL 2001*

A recent debate…

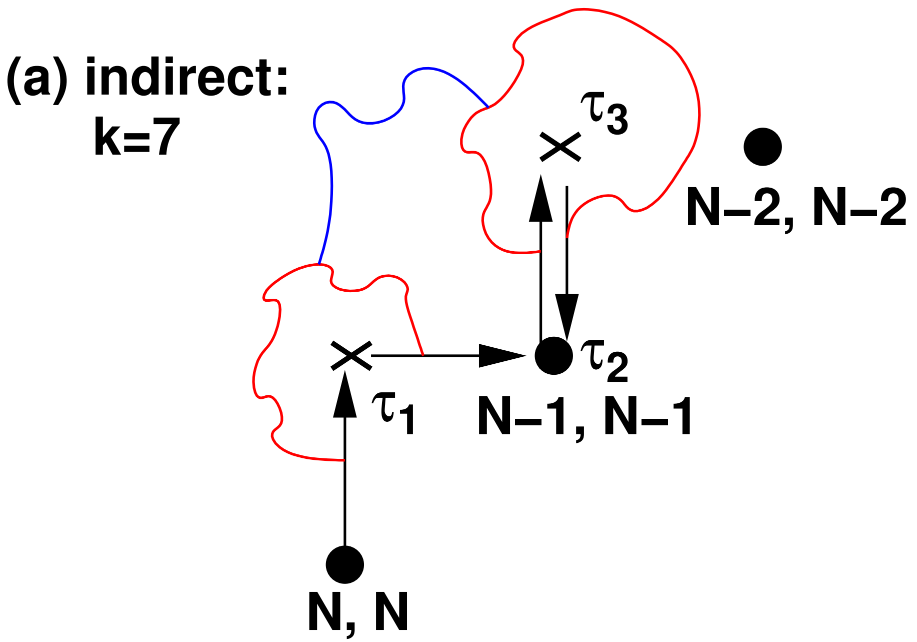
- CST theory of Grifoni et al. based on uncontrolled approximations
- No indication for CST power law scaling in expansions around noninteracting limit

Nazarov & Glazman, PRL 2003,
Gornyi et al., PRB 2003,
Meden et al., PRB 2005

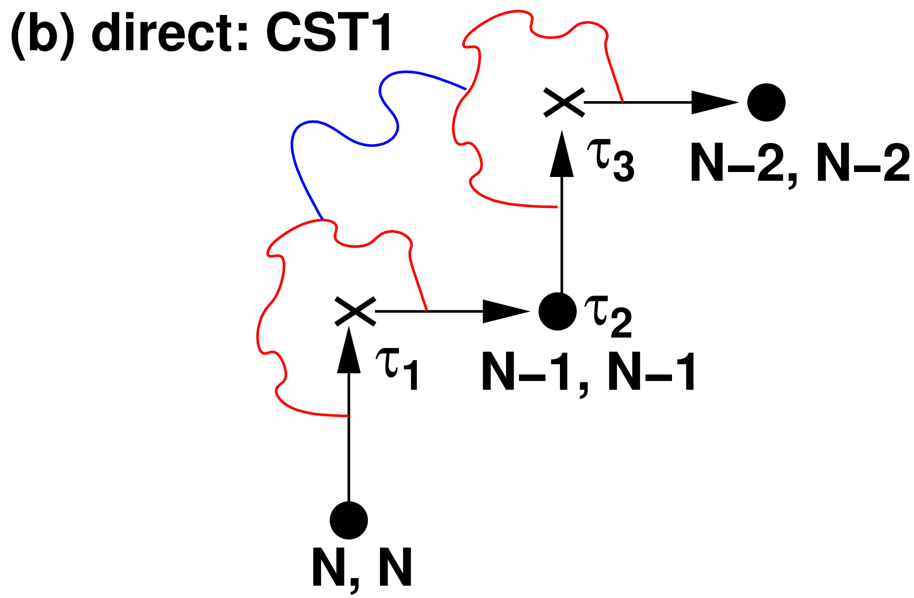
- What is going on?
  - Master equation approach: systematic evaluation of higher order rates
  - Numerically exact dynamical QMC simulations

Fourth-order rate contributions

Thorwart et al. PRB 2005



Renormalization of dot lifetime

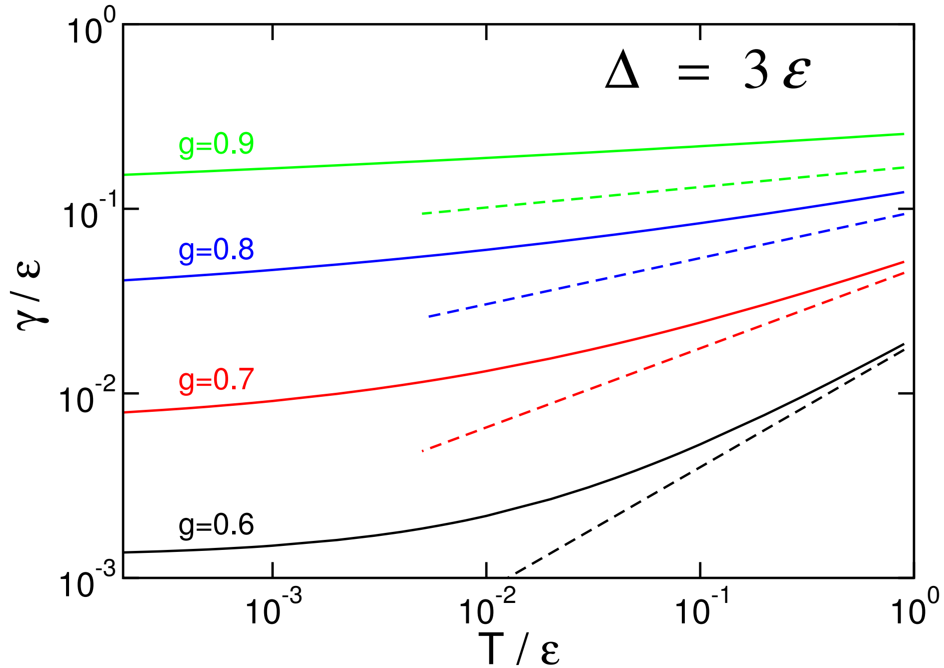


Hop from left to right without cutting the diagram on the dot:
Correlated Sequential Tunneling (CST)

Wigner-Weisskopf regularization

- CST rates *per se* divergent need regularization
- Such processes important in bridged electron transfer theory *Hu & Mukamel, JCP 1989*
- Systematic self-consistent scheme:
  - First assume finite lifetime on dot to regularize diagrams
  - Then compute lifetime self-consistently using all (up to 4th-order) rates

Self-consistent dot (inverse) lifetime

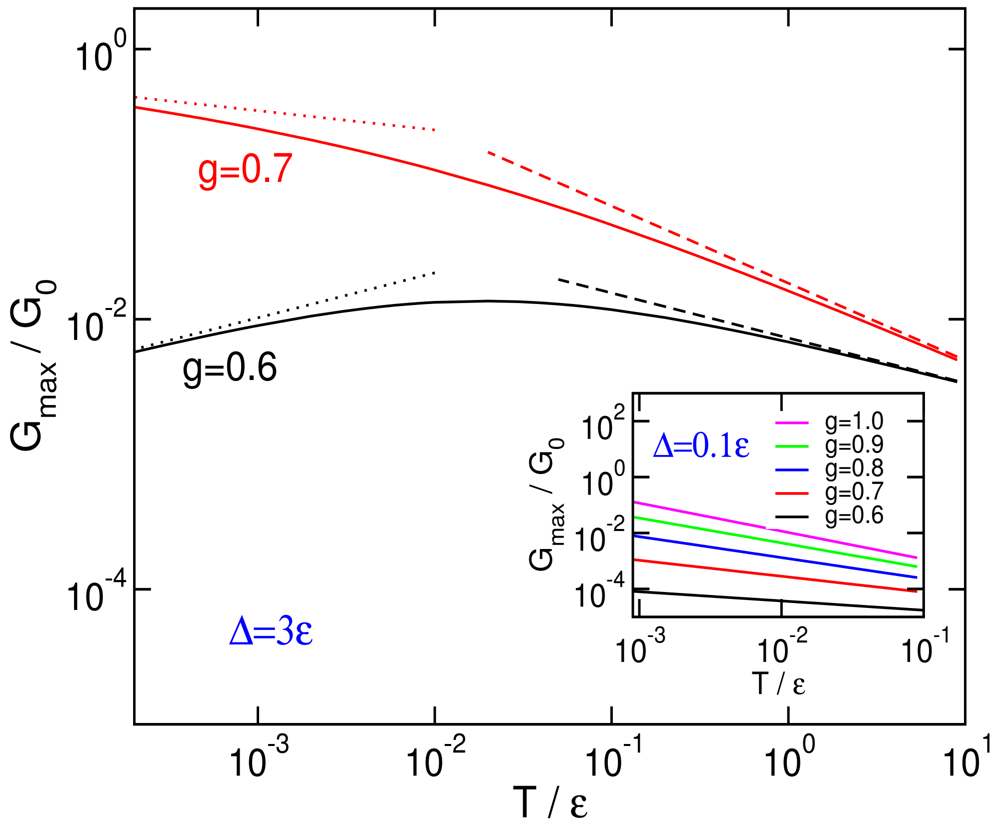


ε : level spacing on dot

Detailed calculation shows:

- CST processes unimportant for high barriers
- CST processes only matter for strong interactions
- Crossover from usual sequential tunneling (UST) at high T to CST at low T

Peak conductance from Master Equation



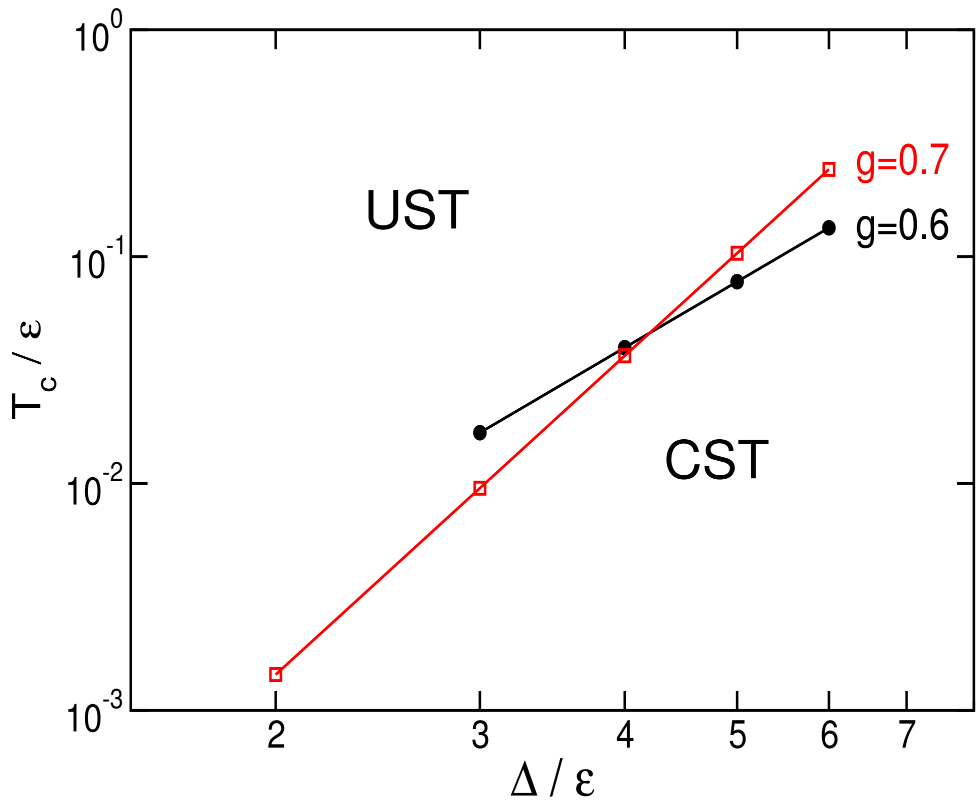
$$G_{\max} \approx \frac{e^2}{h} \frac{\Gamma_{UST}^2}{T} \propto T^{-3+2/g}$$

- Crossover from UST to CST for both interaction strengths
- Temperature well below level spacing ε
- Incoherent regime, no resonant tunneling
- No true power law scaling
- No CST for high barriers (small Δ)

Crossover temperature separating UST and CST regimes

- CST only important for strong e-e interactions
- No accessible *T* window for weak interactions
- At very low *T*: coherent resonant tunneling

⟶ CST regime possible, but only in narrow parameter region

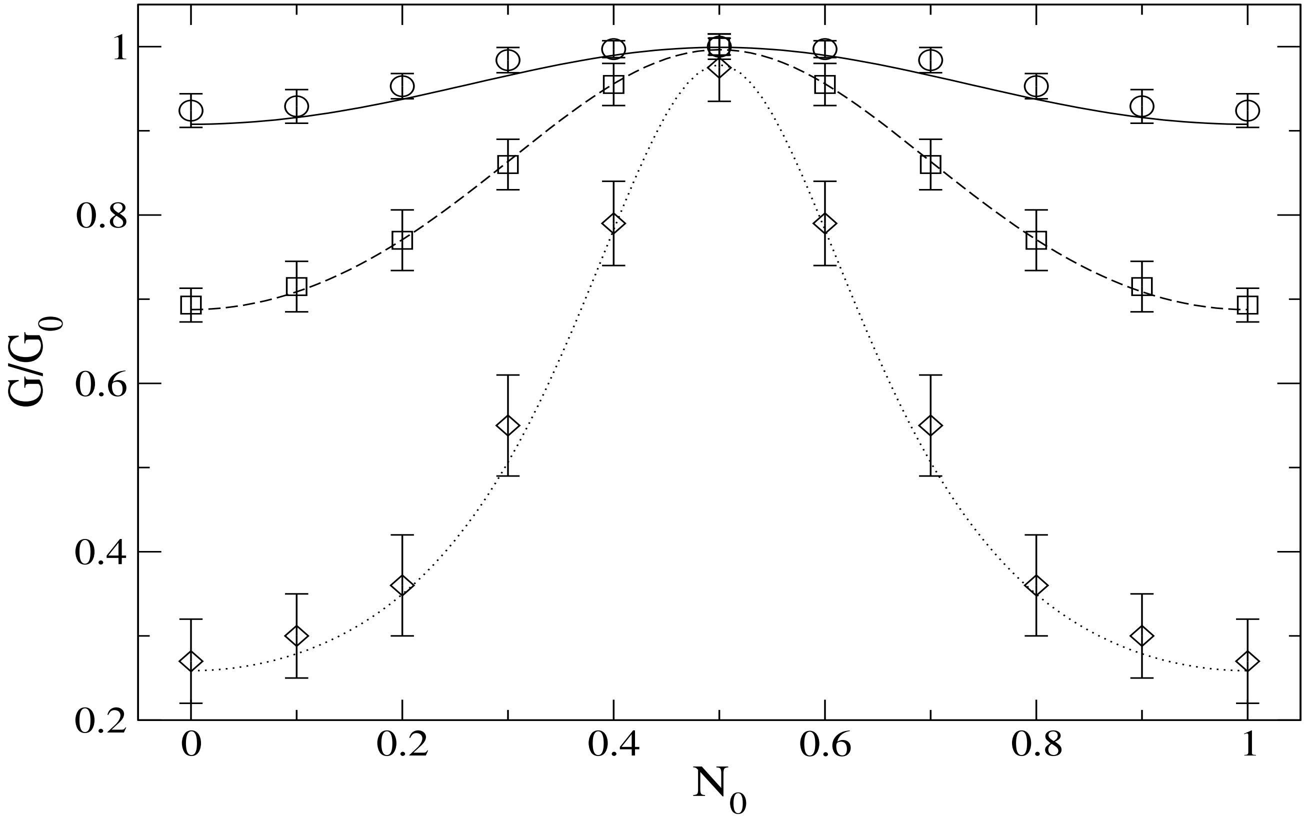


Real-time QMC approach

- Alternative, numerically exact approach, applicable also out of equilibrium
- Does not rely on Master equation
- Map coupled Quantum Brownian motion problem to Coulomb gas representation
- Main obstacle: Sign problem, yet asymptotic low-temperature regime can be reached

Hügle & Egger, EPL 2004

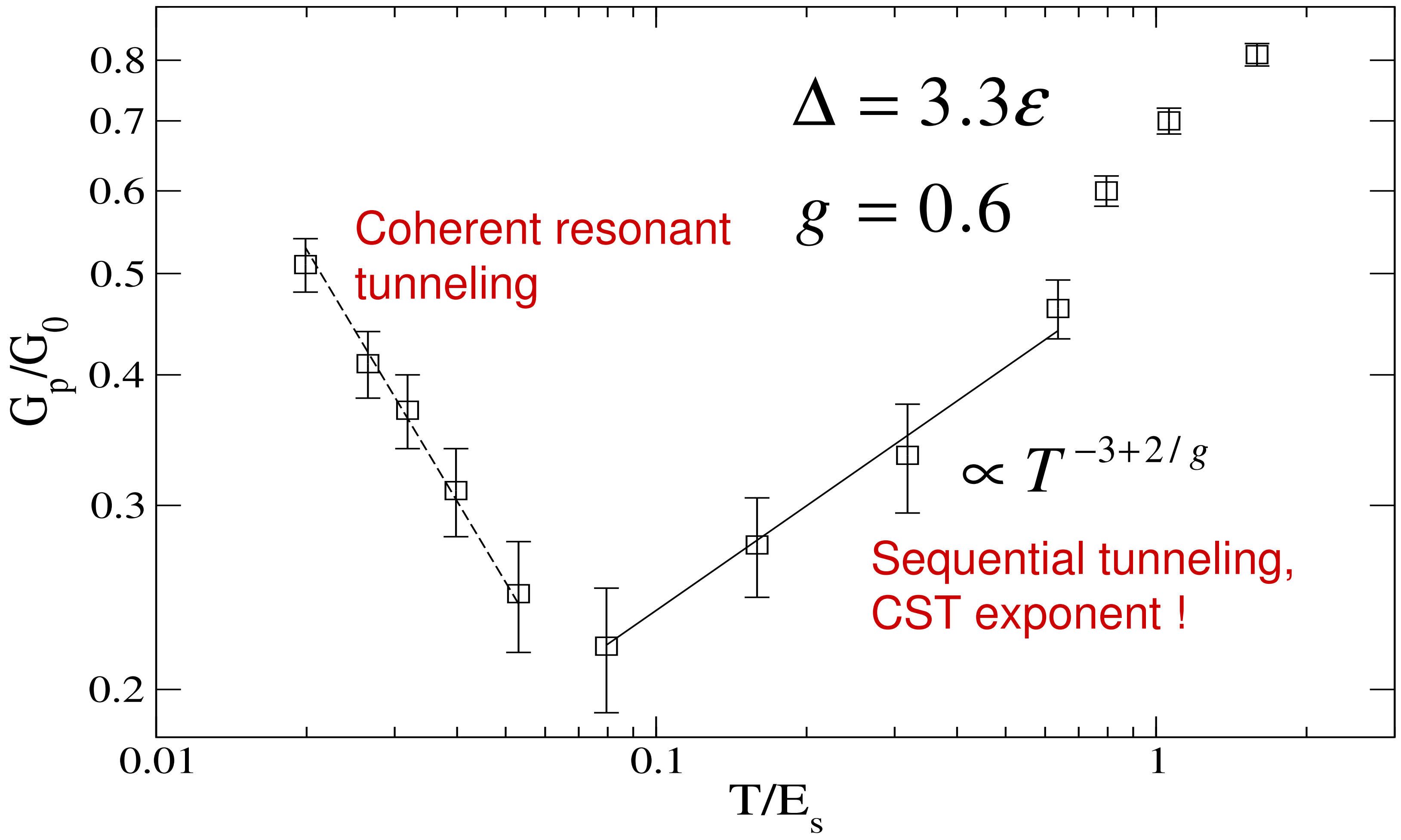
Check QMC against exact *g=1* result



QMC reliable and accurate

Peak height from QMC

Hügle & Egger, EPL 2004



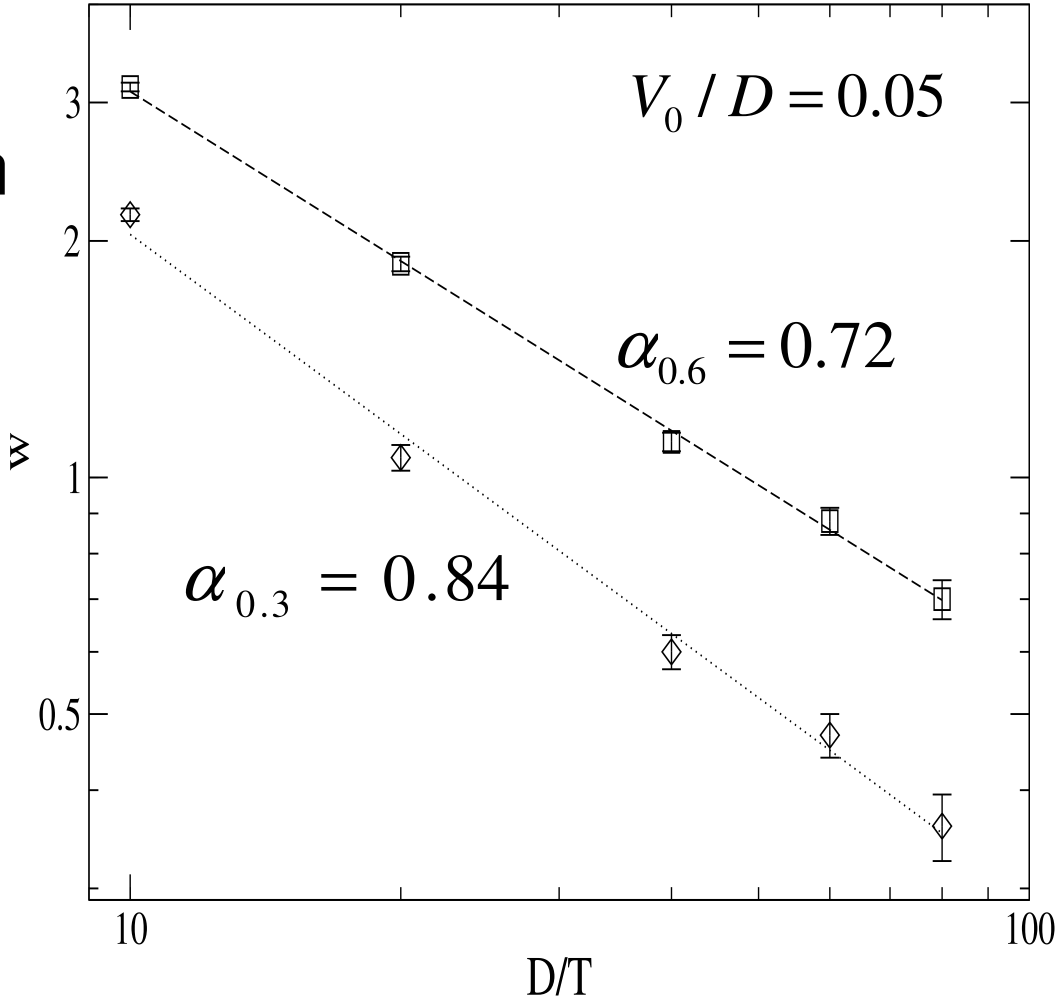
CST effects seen in simulation…

Strong transmission behavior

- For $k_B T / D > 0.01$: *g=1* lineshape but with renormalized width

$$w = w_g(T) \propto T^{\alpha_g}$$

- Fabry-Perot regime, broad resonance
- At lower *T*: Coherent resonant tunneling



Coherent resonant tunneling

Low T, arbitrary transmission:
Universal scaling

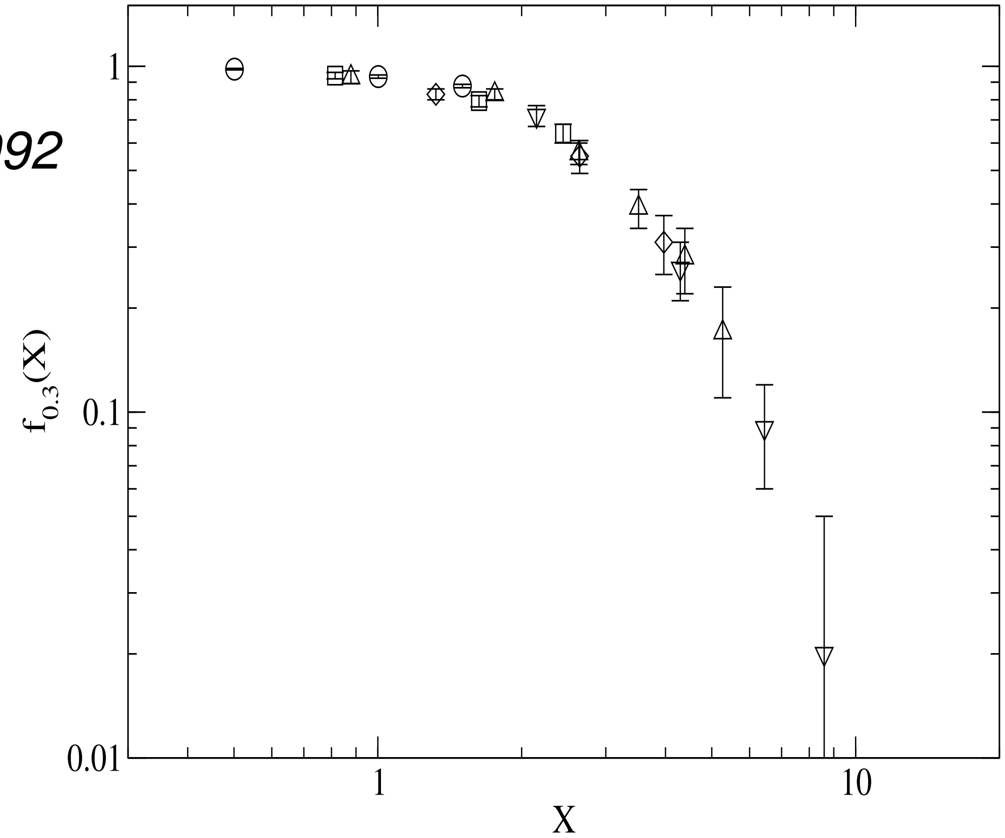
Kane & Fisher, PRB 1992

$$G/G_0 = f_g(X)$$

$$X = T^{g-1}\left|N_0 - 1/2\right|$$

$$f(X \to 0) = 1 - X^2$$

$$f(X \to \infty) \propto X^{-2/g}$$



Conclusions

- Pronounced effects of electron-electron interactions in tunneling through a double barrier
- CST processes important in a narrow parameter regime, but no true CST power law scaling:
  - Intermediate barrier transparency
  - Strong interactions & low T
- Results of rate equation agree with dynamical QMC
- Estimates of parameters for Delft experiment indicate relevant regime for CST