Theoretical Quantum Optics and Quantum Information (SS 2016)

Exercise IV (discussion: 06.06.16)

1. Coherence Property of Light

Calculate the second-order correlation

\[ g^{(2)}(0) = \frac{\langle \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b} \rangle}{\langle \hat{b}^\dagger \hat{b} \rangle^2} \]

for the

(a) coherent state \( |\alpha\rangle \),

(b) Fock state \( |n\rangle \),

(c) Squeezed state \( |\alpha, r\rangle \).

Recall that for squeezed states, the mean photon number is given by \( \bar{n} = \langle \hat{n} \rangle = |\alpha|^2 + \sinh^2 r \), and its variance is equal to \( (\Delta n)^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 = |\alpha \cosh r - \alpha^* e^{2i\varphi} \sinh r|^2 + 2 \cosh^2 r \sinh^2 r \).

2. Photon Statistics

Find the second-order correlation function

\[ g^{(2)}_t(0) = \frac{\langle \hat{b}(t) \hat{b}(t) \hat{b}(t) \hat{b}(t) \rangle}{\langle \hat{b}(t) \hat{b}(t) \rangle^2} \]

of the light produced via parametric down conversion from

(a) Vacuum state

an initial vacuum state. Show that light exhibits bunching \( g^{(2)}_t(0) > 1 \).

(b) Coherent state

an initial coherent state. Show that in this case the output will exhibit antibunching \( g^{(2)}_t(0) < 1 \). You can restrict your analysis to the weak interaction case, i.e. for \( \chi t \ll 1 \), where \( \chi \) is the susceptibility. Note that \( \hat{D}(\alpha) \hat{S}(\varepsilon) = \hat{S}(\varepsilon) \hat{D}(\gamma) \), where \( \gamma = \alpha \cosh r + \alpha^* e^{2i\varphi} \sinh r \).
3. Shannon’s noiseless coding theorem.

Consider an alphabet composed of five letters \( \{a, t, f, o, i\} \) and two languages where the frequency of the letters are given by:

<table>
<thead>
<tr>
<th>Letter</th>
<th>Language 1 Prob. distribution ( p_1 )</th>
<th>Language 2 Prob. distribution ( p_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.3</td>
<td>0.07</td>
</tr>
<tr>
<td>t</td>
<td>0.21</td>
<td>0.01</td>
</tr>
<tr>
<td>f</td>
<td>0.2</td>
<td>0.65</td>
</tr>
<tr>
<td>o</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>i</td>
<td>0.11</td>
<td>0.12</td>
</tr>
</tbody>
</table>

(a) Calculate the average minimum number of bits asymptotically needed to encode a letter in the two languages of the table above.

(b) Consider the following encoding:

\[
a \rightarrow 0001 \quad t \rightarrow 001 \quad f \rightarrow 00001 \quad o \rightarrow 00000 \quad i \rightarrow 1.
\]

What is the average number of bits used for encoding a letter in the two languages?

(c) Create for each language a new encoding which uses on average less bits than the one proposed above and calculate the average number of bits required by your encoding.

(d) Consider the following encoding:

\[
a \rightarrow 0 \quad t \rightarrow 1 \quad f \rightarrow 00 \quad o \rightarrow 11 \quad i \rightarrow 10.
\]

Calculate the average number of bits needed for encoding a letter in language 1. This encoding would need less bits than an optimal encoding (see 3a). What is the problem with this encoding?