1. **Entanglement Swapping**

In the lecture the entanglement swapping protocol was presented. Show that a Bell measurement on particles Z and A in the state

\[ |\psi^{\text{total}}\rangle = |\psi\rangle_{YZ} \otimes |\psi^{-}\rangle_{AB} = [\alpha|01\rangle + \beta|10\rangle]_{YZ} \otimes \frac{1}{\sqrt{2}} ([01] - |10\rangle)_{AB}, \]

followed by an appropriate rotation of the particles B, results in the desired state \( |\psi\rangle_{BY} \), up to a global phase. Which rotation of B is needed for which outcome of the Bell measurement?

2. **CNOT in a rotated basis**

Show that:

3. **Creating GHZ states with quantum networks**

Generalize the “Bell state network”

\[
\begin{align*}
|x\rangle & - H \\
|y\rangle & - \bigoplus \\
& \bigg| \text{[Bell]} \bigg|
\end{align*}
\]

presented in the lecture to a network creating (and measuring) the eight \( n = 3 \) qubit GHZ states

\[ |\text{GHZ}_i\rangle_{\pm} = \frac{1}{\sqrt{2}} (|i\rangle \pm |2^n - 1 - i\rangle), \quad i = 0, 1, \ldots, n, \]

where numbers in the kets on the right hand side are in binary notation.\(^1\)

\(^1\) For instance, \( |\text{GHZ}_2\rangle = 1/\sqrt{2}(|2\rangle + |5\rangle) = 1/\sqrt{2}(|010\rangle + |101\rangle). \)
4. *The Deutsch-Jozsa Algorithm*

Start with $|0\rangle |\varphi\rangle$ and calculate the result of the Deutsch-Jozsa algorithm with the quantum network shown below. How does one get the information about whether the function $f(x)$ is balanced or not? Here $|\varphi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ and $U_f : |x\rangle |y\rangle \mapsto |x\rangle |f(x) \oplus y\rangle$.

![Quantum Network Diagram]

For Fun: What is the smallest number formed twice by summing two cubes - this is how I remember my Geheimzahl number!