Quantum Optics and Quantum Information SS 2006
Exercise VI (Wednesday 17.05.06)

I. COHERENCE PROPERTY OF THE LIGHT

Calculate the second-order correlation

$$g^{(2)}(0) = \frac{\langle \hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}\rangle}{\langle \hat{a}^{\dagger}\hat{a}\rangle^{2}}$$

for
1.coherent state $|\alpha\rangle$
2.Fock state $|n\rangle$.
3. Squeezed state $|\alpha, r\rangle$

II. PHOTON STATISTICS

Find the second-order correlation function

$$g^{(2)}(0) = \frac{\langle \hat{a}^{\dagger}(t)\,\hat{a}^{\dagger}(t)\,\hat{a}(t)\,\hat{a}(t)\rangle}{\langle \hat{a}^{\dagger}(t)\,\hat{a}(t)\rangle^{2}}$$

of the light produced from
1. an initial vacum state via parametric downconversion . Show that the light exhibits bunching $\left(g^{(2)}(0) > 1\right)$.
2. an initial coherent state. Show that in this case the output light is antibunched.

III. BEAM–SPLITTERS

A 50-50 beam splitter transforms incoming mode $\hat{a}$ and $\hat{b}$ to the outgoing operators $\hat{c}$ and $\hat{d}$

$$\hat{a} = \frac{\hat{c} + \hat{d}}{\sqrt{2}},$$
$$\hat{b} = \frac{\hat{c} - \hat{d}}{\sqrt{2}},$$

where $\left[\hat{c}, \hat{d}\right] = 0$, $\left[\hat{c}^{\dagger}, \hat{d}\right] = 0$.

What happens when a coherent state $|\alpha\rangle$ passes through this beam-splitter?

Investigate the statistical properties of outgoing fields i.e. $\hat{c}$ and $\hat{d}$.

Hint: The beam splitting process is probabilistic at the level of individual photons. Hence any state of the light is randomized by the beam splitter. The coherent states are very special, they behave like classical waves at the beam-splitter.

After the beam splitter the state is

$$|\alpha\rangle_a \otimes |0\rangle_b \Rightarrow \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}\frac{1}{\sqrt{n!}}\left(\frac{\hat{c}^{\dagger} + \hat{d}^{\dagger}}{\sqrt{2}}\right)^{n}|0\rangle_c \otimes |0\rangle_d$$